MULTI-LEVEL ANALYSIS OF DYNAMIC PORTFOLIO FORMATION: CE COUNTRIES

MICHAEL PRINC

Abstract

The paper focused on measuring efficiency of investment strategies and portfolio optimization based on dynamic portfolio creation using global minimum variance approach. The paper analyses DCC GARCH model, which was employed in order to obtain conditional correlation matrices. The analysis includes a composition of various portfolios and focuses on a comparison between micro- and macro-level portfolios in a region of central European countries. List of portfolios includes global minimum variance portfolios (GMV) and newly proposed least correlated assets portfolios (LCA). Performance of portfolios constituted from individual shares and market indexes showed that dynamic form of portfolio optimization is an efficient tool in profit maximization and volatility minimization. The study shows that there is a potential for improvements of proposed methods. LCA portfolio formation showed that the number of parameters could be effectively lowered without a loss profit.

Keywords

Dynamic modelling, GMV, portfolio selection, regional analysis.

Introduction

Reasoning about possible investment opportunities has usually two main objectives. Investors have usually a desire to maximize profits, but on the other hand they also want to keep a volatility of the portfolio as low as possible. Following work is a practical econometric analysis applicable to portfolio and financial management, economic valuation or investment planning.

In a case of the technical analysis of stock markets, the goals are usually achieved by portfolio diversification, which offers lowering of unsystemic risk as in Markowitz (1952), Markowitz (1995) or DeMiguel et al. (2009).

We can find several studies focused on portfolio analysis, which are either dedicated to various financial indexes (e.g. Antzoulatos – Tsoumas (2010), Banerjee - Doran - Peterson (2007), Liao - Chou (2013) or Konno - Kobayashi (1997)) or individual stocks (e.g. Balcilar et al. (2013), Nanda et al. (2010) or Lan et al. (2012)). Following text focuses on synergic analysis, which combines indexes with individual stocks in order to offer a more complex overview. We would like to offer insight into dynamic portfolio formation methods applied on data series from Central Europe based on a multi-level analysis.

The multi-level analysis consists from two different points of view. The low-level or rather micro-level analysis is devoted to portfolio diversification strategies focused on particular firms, while index-level or so called macro-level analysis deals with diversification based on stock market indexes in the international framework as in Aiello - Chieffe (1999) or Rapach – Wohar (2009). The multi-level analysis is proposed in order to compare the effectiveness of portfolio diversification strategies focused either on indexes or individual shares. Thanks to the employed aggregation of individual shares on a national level all outcomes are directly comparable, which further increase the scientific impact of the work in a context of the researched region.

Consistent with the widely used CAPM model, proposed by Sharpe (1964) or Lintner (1965), market indexes serve as the best proxies of actual development of markets or even whole economies and thus we regard it as appropriate to speak about macro-level point of view, when market indexes are analysed. The micro-level portfolios consist of most liquid stock traded on analysed markets; they are constructed in order to reveal potential of individual shares focusing on actual firm developments, which is consistent with a microeconomic type of analysis.

The main objective of the work is a description of possible strategies, which can investor use to optimize portfolio in terms of dynamic analysis based on DCC GARCH model proposed by Engle (2002).

Following optimization techniques are inspired by Markowitz (1952) and proposed approaches to the optimum portfolio selection problem. The full list of used strategies includes equally weighted (EW), global minimum variance (GMV), unbounded (or leveraged) global minimum variance (UGMV) and least correlated assets (LCA) portfolios. The main advantage of the newly proposed LCA portfolio is that it eliminates a possible problem with hardly predictable volatility components. This offer a further possibility to test a hypothesis whether the volatility component included in the portfolio selection method can improve or downgrade actual performance.

While these portfolio optimization methods are usually used in a standard static mean-variance approach, the paper applies them into a new dynamic framework, which is a totally new method never used in the central European region.

The work is trying to confirm hypothesis that proposed dynamic portfolio formation methods can lead to improved returns or lower volatility. The hypothesis is tested for both micro- and macro-level portfolios in a central European region, in order to compare the effectiveness of diversification methods on various levels.

In this study word dynamic is understood as continuously changing of evolving in terms of dynamic econometric models. This is in a slight contrast to the traditional understanding of connotation “dynamic portfolio”, which usually means “more risky”. In this case the purpose of dynamically created portfolios is rather to decrease overall volatility with respect to maintained level of returns, which is consistent with a modern portfolio theory.

1. Literature Review

As was mentioned during the introduction, there are usually two types of studies. The first type of studies is devoted to various financial indexes (e.g. Antzoulatos – Tsoumas (2010), Banerjee - Doran - Peterson (2007), Liao - Chou (2013) or Konno - Kobayashi (1997)), while the second type is focused on individual stocks (e.g. Balcilar et al. (2013), Nanda et al. (2010) or Lan et al. (2012)). We would like to include a synergic point of view, which includes both perspectives, thus we would like to offer a multi-level approach, which combines individual stocks with respective stock indexes.

The work also focuses on various portfolio strategies. The equally weighted (EW) portfolio selection was employed e.g. in Solnik (1974) or Solnik (1991) which described portfolio performance on a basis of the classical covariance matrix approach. In this case it will serve as a kind of benchmark, which can indicate lowest bound of performance attainable by relatively simple diversification strategy.

In our study we will research GMV portfolios, which optimize investment with respect to the minimized variance (for further details see Bodnar - Okhrin (2013) or Yilmaz (2010)).

In a comparison to traditional mean-variance approach initially established in Markowitz (1995), methods used in following study focus primarily on a variance of portfolio, dynamic correlations and respective dynamic covariances.

The study is analysing similar method like in Yilmaz (2010), which proved that the DCC GARCH model can be successfully used as an enhancing tool for portfolio creation, but a focus was kept only on Istanbul Stock Exchange, which was a single domestic market. In this case primary objective is to analyse more countries and stock markets emphasizing international dimension of a possible portfolio formation as in previously mentioned index-based studies, which is a new and inspiring approach in the central European region.

Following study is building its methodology on previous successful identification of GARCH processes in financial series. The existence of respective GARCH processes was proved e.g. in Bollerslev (1986) in a case of well-developed markets or in Vošvrda - Žikeš (2004) in a case of central European markets.

A dynamic approach offers more possibilities than classic portfolio measures like in Markowitz (1952), where a simplified and static approach was employed. Proposed dynamization can show an evolution of underlying processes and reveal possible diversification opportunities.
A dynamic model using conditional correlation/covariance matrix based on GARCH modelling was developed by Engle (2002), who designed the DCC GARCH model and started also an application to portfolio formation processes. The model was justified and widely used in many interesting studies e.g. Engle - Sheppard (2001) or Cappiello et al. (2006). The paper is trying to extend previous methods and to offer more methodological tools how to interpret the outcomes of dynamic models.

2. Methodology

2.1 GARCH Model

The generalized autoregressive conditional heteroskedasticity model proposed by Bollerslev (1986) could be briefly described either as an error variance model incorporating autoregressive moving average or as a generalization of ARCH model by Engle (1982). The GARCH model approach allows for an empirical assessment of the relationship between risk and returns in a setting that is consistent with the characteristics of a leptokurtosis and a volatility clustering observed in the stock market data series. In the univariate GARCH model, proposed in Bollerslev (1986), we assume conditional variance is defined as in (1).

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-1}^2, p \geq 0, q > 0, i > 0 \tag{1}$$

$$y_t = \mu + \varepsilon_t \tag{2}$$

$$\varepsilon_t = \sigma_t z_t \tag{3}$$

$$z_t \sim iid(0,1) \tag{4}$$

In equation (1) it is necessary to ensure non-negative variance σ_t^2. We can derive vector ε_t from the equation (2) and following conditions (3) and (4). The model used in the study is GARCH (1,1), which has a following form:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{5}$$

In order to ensure stationarity of the process it is necessary that $\alpha + \beta < 1$.

2.2 DCC GARCH Model

The dynamic conditional correlation multivariate GARCH model, which will be used as a tool for derivation of actual correlations, was originally designed in Engle (2002). The definition is as follows:

$$r_t \mid \Psi_{t-1} \sim N(0, H_t) \tag{6}$$

$$H_t = D_t R_t D_t \tag{7}$$

$$D_t^2 = diag\{\omega_i\} + diag\{\kappa_i\} r_{t-1} r'_{t-1} + diag\{\gamma_i\} D_{t-1}^2 \tag{8}$$

$$\varepsilon_t = D_t^{-1} r_t \tag{9}$$

$$Q_t = S(\iota\iota' - A - B) + A\varepsilon_{t-1}\varepsilon'_{t-1} + BQ_{t-1} \tag{10}$$

$$R_t = diag\{Q_t\}^{-1} Q_t diag Q_t^{-1} \tag{11}$$

At first it is necessary to assume normality and stationarity of the underlying process as described in (6), while denoted matrix H_t can be further decomposed using a method described in (7). An equation (8) expresses that each subset follows a univariate GARCH process as defined in (1) (or (5) in a case of applied GARCH (1,1) model).

The equation (9) describes behaviour of residual terms with respect to input data and describes its relation to analyzed returns. Finally relations (10) and (11) describe the matrix composition necessary for the estimation and iteration processes. When the assumption of normality in (6) is not fulfilled, the estimator could be marked only as a quasi-maximum likelihood estimator (QMLE). The log likelihood function for the proposed estimator is described as follows in a relation (12)[1]:

$$\log(L) = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log(D_t(r_t' D_t^{-1} D_t^{-1} r_t - \varepsilon_t'\varepsilon_t + \log(R_t(\varepsilon_t' R_t^{-1}\varepsilon_t)\right), \tag{12}$$

In order to maximize described parameters it is necessary to fulfil conditions from (6) to (11). One of main advantages of the DCC GARCH model is that the log-likelihood function can be divided into two separate parts (13) and (14), where relation to individual GARCH processes is resolved and can be maximized separately:

$$\log(L)(\theta,\varphi) = \log(L_V)(\theta) + \log(L_C)(\theta,\varphi) \tag{13}$$

[1] for further details see Engle (2002) or Pelagatti - Rondena (2004)

$$\log\left(L_V\right)(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{n}\left(\log\left(2\pi\right)+\log\left(h_{it}\right)+\frac{r_{it}^2}{h_{it}}\right) \tag{14}$$

The equation (14) reflects volatility, which is a sum of individual univariate GARCH log-likelihood functions. This step urges a need of prior estimations of all involved univariate GARCH models. A second term of (13) describes conditional correlation parameters, which are also maximized individually, which results to relatively lower needs for computational power. Final two stage estimation in equations (15) and (16) describes a relation of maximized parameters.

$$\hat{\theta} = \arg\max\left\{L_V(\theta)\right\} \tag{15}$$

$$\max_{\phi}\left\{L_C(\hat{\theta},\phi)\right\} \tag{16}$$

For further information about estimation of proposed parameters see also Engle (2002), Pelagatti - Rondena (2004) or Princ (2010).

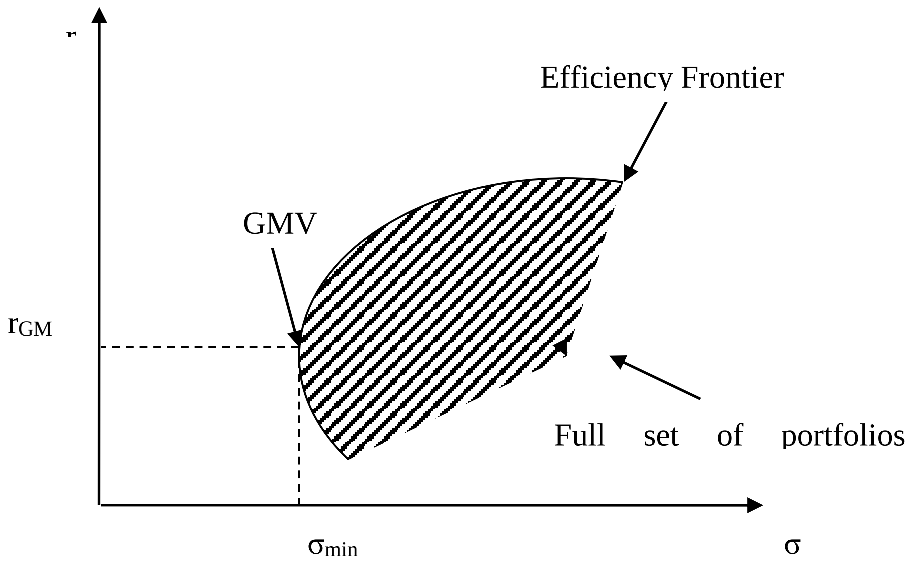
2.3 Diversification Strategies

Investment strategies may differ; they are dependent on initial assumptions, which comply with a goal of the investor. Further text uncovers differences between a simple benchmark method represented by EW portfolio, which minimizes effort exercised in the optimization process, and other portfolios, which use re-weighting techniques.

The DCC GARCH model provides conditional correlation/covariance matrices, which are necessary to achieve the dynamic analysis of all following approaches, namely GMV, UGMV and LCA portfolios. While we compute dynamic conditional correlations and variances, we are able to do portfolio optimization process at every day of researched sample, thus we include time index *t* to all following mentioned equations.

A general example of GMV portfolio in the context of the Capital Market Line (CML) model is depicted in the Figure 1.

Figure 1: GMV Representation in Capital Market Line Model



The variance of portfolios is computed on a basis of estimated of various processes and computed correlation/covariance matrices, which were estimated through DCC GARCH models. The concrete definition of a variance of assumed portfolios is as follows:

$$\sigma_{p_t}^2 = \sum_{i=1}^{n} w_{i_t}^2 \hat{\sigma}_{i_t}^2 + \sum_{i=1}^{n} \sum_{j=1 i \neq j}^{n} \hat{\rho}_{ij_t} w_{i_t} w_{j_t} \hat{\sigma}_{i_t} \hat{\sigma}_{j_t} \tag{17}$$

The formula is a classic form of a portfolio variance mentioned in e.g. Elton - Gruber (1991). Specific values of weights w_{i_t}, w_{j_t} are defined in the next part, which is devoted to specific investment strategies. In this general framework we can state that variance components $\hat{\sigma}_{i_t}^2$ can be estimated by GARCH (1,1) model and correlation components $\hat{\rho}_{ij_t}$ can be estimated by the DCC GARCH model.

2.3.1 Equally Weighted Portfolio

The equally weighted portfolio (sometimes also marked as 1/n or 1-n portfolio) is a common diversification tool used in portfolio theory. Because the method presumes simple assumptions, it is plausible to mark it as a benchmark for more sophisticated methods. Any other way achieving lesser variance or possible higher yield can be marked as more effective as a benchmark and thus achieving a higher bound of portfolio effectiveness. In an opposite way a portfolio with higher values of volatility or lower yield could be marked as ineffective.

The weights for EW portfolio are defined in a following way:

$$w_t = [1/n, 1/n, \ldots, 1/n] \tag{18}$$

$$w_t'\mathbf{1} = 1 \tag{19}$$

$$\mathbf{1} = (1, 1, \ldots, 1)' \tag{20}$$

2.3.2 Global Minimum Variance Portfolio

While all efficient portfolios lie on the efficient frontier, GMV is the one that is exactly at the beginning of perceived frontier. Ex ante GMV has the lowest possible volatility amongst other efficient portfolios, but it does not mean that it should achieve better performance in a term of higher returns, because solely the variance is optimized (see Figure 1). The mathematical construction of the GMV portfolio can be expressed in a following way[2]:

$$min_{w_{gmv}} = \frac{1}{2} w_t' \Sigma_t w_t \tag{21}$$

$$w_t = \frac{\Sigma_t^{-1}\mathbf{1}}{\mathbf{1}'\Sigma_t^{-1}\mathbf{1}} \tag{22}$$

$$w_t'\mathbf{1} = 1 \tag{23}$$

$$\Sigma_{GMV_t} = \begin{bmatrix} \hat{\sigma}_{1_t}^2 & \hat{\rho}_{12_t}\hat{\sigma}_{1_t}\hat{\sigma}_{2_t} & \cdots & \hat{\rho}_{1n_t}\hat{\sigma}_{1_t}\hat{\sigma}_{n_t} \\ \hat{\rho}_{12_t}\hat{\sigma}_{2_t}\hat{\sigma}_{1_t} & \hat{\sigma}_{2_t}^2 & & \vdots \\ \vdots & & \ddots & \vdots \\ \hat{\rho}_{1n_t}\hat{\sigma}_{n_t}\hat{\sigma}_{1_t} & \cdots & \cdots & \hat{\sigma}_{n_t}^2 \end{bmatrix} \tag{24}$$

We can assume that we have an opportunity to invest in *n* assets; vector of weights *w* can be derived from equation (22) or when the minimization is employed in equation (21). It is also necessary to assume that when we add all individual weights together, the sum is equal to 1 as in equation (22); this is consistent with a previous condition in equation (19) of EW portfolio. Equation (24) shows a definition of matrix Σ_{GMV_t}, which is a basis for further estimations used in GMV portfolio optimizations defined in equation (21) and (22). Input values of Σ_{GMV_t} are estimated with DCC GARCH ($\hat{\rho}_{ij_t}$) and GARCH (1,1) ($\hat{\sigma}_{i_t}^2$ and $\hat{\sigma}_{i_t}$) models.

[2] For further details see Bodnar – Okhrin (2013) or Yilmaz (2010).

In a case of GMV portfolio we also assume that all individual weights[3] are greater or equal to 0. In a case of UGMV portfolio we assume that the condition, which prohibits weights from reach negative values, is released. UGMV portfolios thus include even negative amounts of individual stocks. We can conclude that in a case of UGMV portfolios there are possibilities to include short selling in portfolio strategies, which can result in a leveraged purchase of individual stocks or indexes.

2.3.3 Least Correlated Assets Portfolio

The least correlated assets (LCA) portfolio can be specified in a similar manner as GMV portfolio in (21), (22) and (23) with additionally imposed restrictions on equation (24). We assume that variance components of Σ matrix are constant during the whole time period as defined in equations (25), which results Σ in matrix described in (26). Newly imposed restrictions offer an opportunity to analyse improvements achieved solely by the DCC GARCH model without any negative effects caused by possibly improper design of evolving variance defined by GARCH (1,1) model.
The restriction is imposed in a part of portfolio variance estimation as specified in (17) or alternatively in Σ matrix, where conditional variances equal to a constant term; only conditional correlations are allowed to be variable as in (26). Input values of Σ_{LCA_t} matrix are estimated by DCC GARCH model.

$$\hat{\sigma}_{i_t}^2 = \sigma_{i_t}^2 = c, \hat{\sigma}_{j_t}^2 = \sigma_{j_t}^2 = c \quad \Rightarrow \hat{\sigma}_{i_t}\hat{\sigma}_{j_t} = \sigma_{i_t}\sigma_{j_t} = c \tag{25}$$

$$\Sigma_{LCA_t} = \begin{bmatrix} c & \hat{\rho}_{12_t}c & \cdots & \hat{\rho}_{1n_t}c \\ \hat{\rho}_{12_t}c & c & & \vdots \\ \vdots & & \ddots & \vdots \\ \hat{\rho}_{1n_t}c & \cdots & \cdots & c \end{bmatrix} \tag{26}$$

3. Data

3.1 Description

All following data series are captured on a daily basis, which offer a suitable environment for investors optimizing their medium-to-long term strategies. The analysis is conducted on a macro-level, which is represented by the analysis of aggregated indexes, and also on a micro-level, which is described by individual shares.

[3] or rather all individual values vector of weights.

Observed data samples start from 31st March 2006 and end at 30th March 2011. This means that also a financial crisis is included into data sample and thus overall performance is rather negative due to worsening economic conditions in the region.

The macro-level analysis consists of BUX for Hungary, PX for Czech Republic and WIG 20 for Poland, while micro-level analysis is made for companies in every state. Slovak market was excluded from the analysis, because its nature is quite different from other described markets[4]. The most liquid Slovak stocks are traded less than normally traded stocks on other CE markets.

We set data samples equal for every country in order to offer similar space for investments and not to discriminate some specific national market, thus we have finally chosen 5 most liquid companies. Data estimated in the routine were calculated in a logarithmic form of returns as is described in a relation (27).

$$r_t = \log\left(p_t / p_{t-1}\right) \times 100, \qquad (27)$$

where p_t stands for a closing value of the computed index. This means that input values of national stock indexes were transformed into daily returns r_t computed as close-to-close value in percentage points.

3.2 Testing Data Series

Testing of stationarity and normality was employed using Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test, Shapiro-Wilk (S-W) test and finally Augmented Dickey-Fuller (ADF) test. For further information we refer to Kwiatkowski et al. (1992), Shapiro - Wilk (1965) and Said-Dickey (1984) or Banerjee (1993).

The KPSS test hypothesis cannot be rejected even at 10% significance level indicating that stationarity of all data series cannot be rejected. Results of ADF test indicate that the existence of unit root can be rejected in favour of alternative that data series do not contain unit roots even on 1 % level. We can thus conclude that all data series can be regarded as stationary and not having unit root. All tests were conducted in both cases of macro-level and also micro-level analyses.

The hypothesis that data series are normally distributed was rejected, this causes that all estimators have to be marked as QMLE instead of maximum likelihood estimators (for further details see Engle (2002)).

[4] Slovak market can be characterized rather as over-the-counter (OTC) market, majority transactions are off-exchange, average liquidity of stocks is very low; thus it is not directly comparable with other markets.

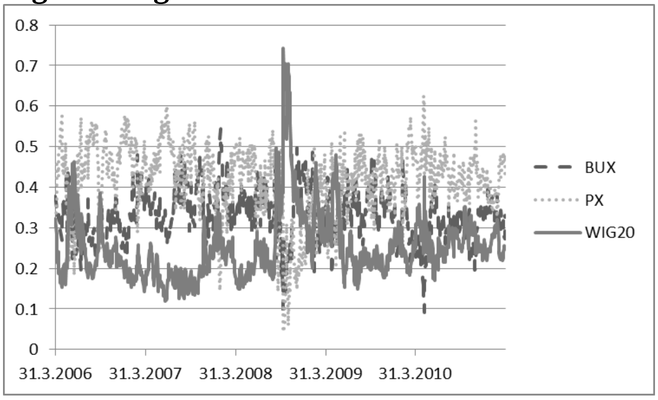
4. Model Estimations

The DCC GARCH model was estimated resulting in a complex correlation/covariance matrix including all the mentioned markets and individual shares. Estimation of GARCH (1,1) models for every time series confirmed the validity of stationarity condition i.e. that it was true that the sum of estimated parameters was lesser than 1 ($\alpha+\beta<1$) for every GARCH (1,1) model, for further details see Bollerslev (1986). All following computations are conducted as in-sample analyses.
Optimization technique used BFGS (Broyden–Fletcher–Goldfarb–Shanno) method using OxEdit, see also Doornik (2007).

4.1 Portfolio Formation Analysis

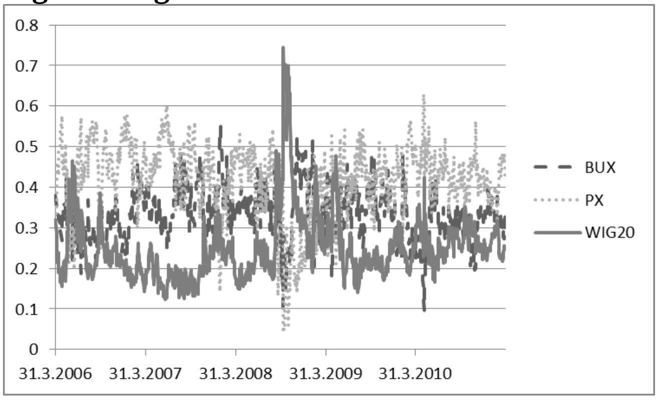
Individual dynamic conditional correlations (DCC) between observed markets and shares were computed using a DDC GARCH methodology computing defined strategies (GMV, UGMV, LCA). The basic difference between GMV and UGMV portfolio is that UGMV approach offers short-selling possibilities.
Because 15 individual weights would be not clearly visible for readers, weights arising from micro analysis were aggregated on a basis of country of origin and marked with a similar symbol as the stock market, where they are traded. This cause that weights are comparable in both types of analyses (micro vs. macro) and also the amount of investment put into specific country is directly comparable.

Fig. 2: Weights of GMV – Micro-level



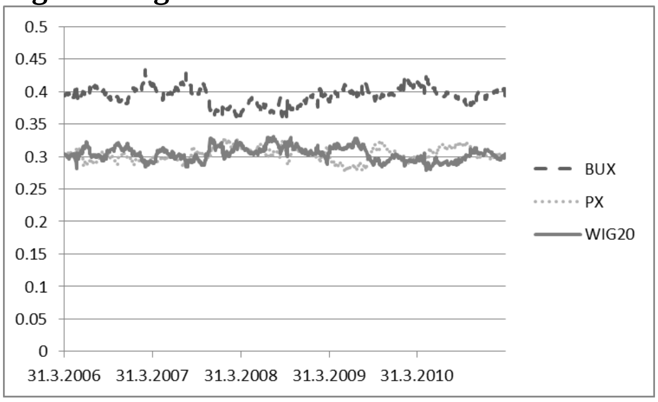
Source: Author's calculations

Fig. 4: Weights of UGMV – Micro-level



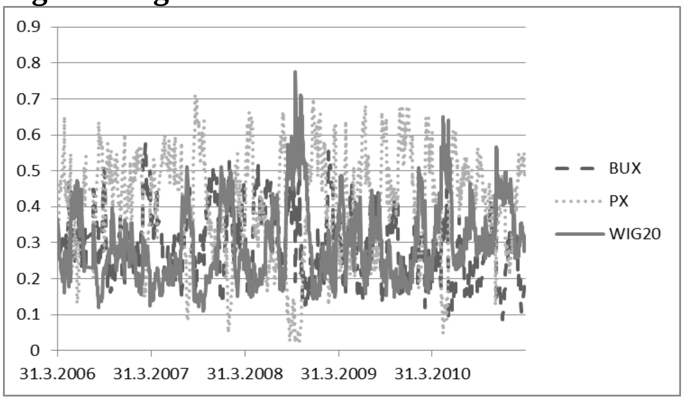
Source: Author's calculations

Fig. 6: Weights of LCA – Micro-level



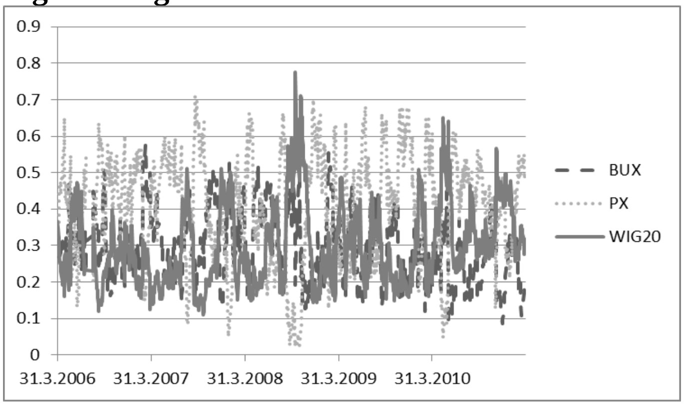
Source: Author’s calculations

Fig. 3: Weights of GMV – Macro-level



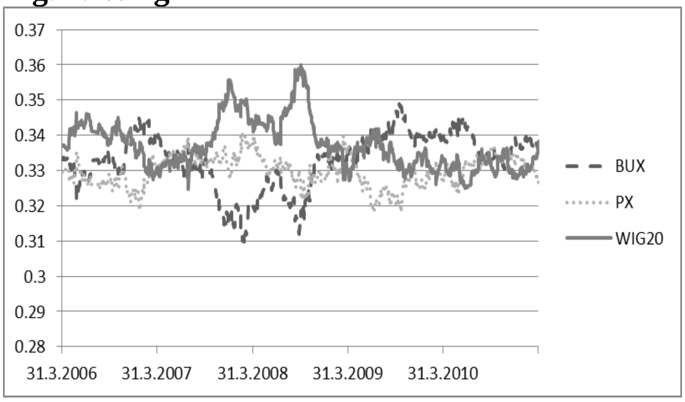
Source: Author’s calculations

Fig. 5: Weights of UGMV – Macro-level



Source: Author’s calculations

Fig. 7: Weights of LCA – Macro-level



Source: Author’s calculations

The only difference between GMV and LCA portfolios is exactly the existence of a volatility component described by GARCH (1,1) model. Figures 2 - 5 show that GMV and UGMV portfolios are not very stable. From Figures 6 and 7 it is clear that volatility component is the reason, why is it so, because LCA portfolios are much more stable and relatively close to the values of EW portfolio. Moreover there is just a mild difference between GMV and UGMV portfolios. This is a very interesting result. Although the weights of individual stocks were even negative, the whole aggregated weights for respective markets are really close in both micro- and macro-level analyses. It is probable that some other methods modelling volatility could improve stability and maybe even profitability of GMV and UGMV portfolios, because LCA portfolios derived solely from DCC GARCH model proved to be less volatile.

4.2 Performance Analysis

Computed weights were used as a basis for further calculations in order to construct portfolios held by a fictional investor. The strategy assumes that the portfolio is held for one day period, then sold and re-weighted according to new information. It means that observed indexes or stocks are bough using data obtained at time *t* and sold at time *t+1*, which means that a data sample is shortened by one observation. The amount received at time $t+1$ is fully reinvested in a same way as at time t. This recursive method is used for the whole data sample.

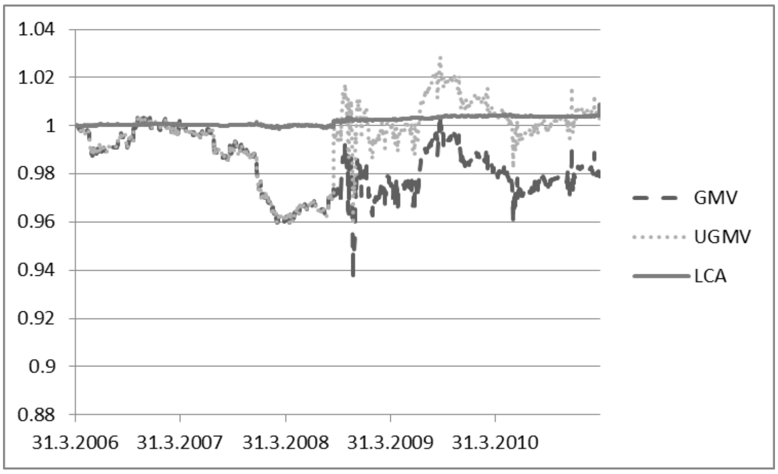
We assume that transaction costs are zero and there is no taxation. We are aware of a fact that these assumptions can decrease actual profitability, but we use these assumptions as a logical basis of our model, which can be extended in future works.

Figures 8 and 9 are describing a relative performance of constructed portfolios (GMV, UGMV and LCA) in a comparison with the benchmark EW portfolio from both micro- and macro-level perspective.

Figure 8: Relative performance of micro-level portfolios

Source: Author’s calculations

Figure 9: Relative performance of macro-level portfolios



Source: Author’s calculations

Both Figures 8 and 9 show that LCA portfolio can be perceived as the best performing investment, which performed better than the EW portfolio benchmark. This means that DCC GARCH can be successfully used in order to optimize portfolios, but on the other hand worse performance of GMV and UGMV portfolios revealed that there is some space for improvements in case of volatility modelling; a usage of GARCH (1,1) in the portfolio optimization had an adverse effect in a comparison to LCA portfolio.

At the time of sudden drops GMV portfolio formation should be probably accompanied by some safety mechanism, which would be capable of recognition of sudden decreases in market prices. It is also probable that in bull markets proposed strategies could achieve much higher returns than the benchmark, which would be fully consistent with Yilmaz (2010). These statements could be explanations of perceived properties, but final statements have to be based on a further research using econometric methods.

The final outcome of the comparison is described in Table 1, basic descriptive statistics are depicted. The Table 1 describes performance statistics on a day-to-day basis.

Table 1: Day-to-Day Performance Comparison of Proposed Portfolio Strategies

	Micro-level Analysis				**Macro-level Analysis**			
	EW	**GMV**	**UGMV**	**LCA**	**EW**	**GMV**	**UGMV**	**LCA**
mean	0.9999	0.9999	0.9999	1	1	1	1	1
skewness	-0.07065	0.04545	0.02563	-0.05886	0.07634	-0.1249	-0.1275	0.07654
kurtosis	5.673	8.542	7.843	4.047	5.51	3.972	3.964	5.466
st. dev.	0.0165	0.01931	0.01945	0.0164	0.01687	0.01596	0.01597	0.01684
minimum	0.898	0.8707	0.8739	0.9034	0.9046	0.9104	0.9104	0.9055
maximum	1.114	1.136	1.135	1.101	1.11	1.079	1.079	1.109

Source: Author's calculations

Values in Table 1 show that LCA portfolio performed better than the benchmark in both scenarios (micro and macro). The mean of daily returns was higher, skewness was less negative or more positive and also standard deviations were lower than the outcomes of the benchmark. Using standard Student's t-statistic[5], proposed in Gosset (1908) and described in equations (28) and (29), improved performance of the LCA portfolio over the benchmark can be marked as statistically significant on 95% confidence level in a case of micro-level analysis.

$$t = \frac{\overline{X}_1 - \overline{X}_2}{S_{X_1X_2} \cdot \sqrt{\frac{2}{n}}} \qquad (28)$$

$$S_{X_1X_2} = \sqrt{\frac{1}{2}(S_{X_1}^2 + S_{X_2}^2)} \qquad (29)$$

where $S_{X_1X_2}$ is pooled standard deviation of both compared groups X_1,X_2; $S_{X_1}^2$, $S_{X_2}^2$ are variances of each group and n is the number of observations.

[5] The Student t-test was testing whether daily returns were statistically different among proposed portfolios during the whole modelled period.

In a case of macro-level analysis LCA portfolio performed also better than the benchmark, alas this difference cannot be marked as statistically significant. The higher profit than in a case of the rest of portfolios is also connected with lower variance in returns than the benchmark. This can be also marked as a further performance improvement of the LCA portfolio over the benchmark.

GMV and UGMV portfolios offer mixed results. Profits are generally lower than in a case of benchmark, but there can be perceived a potential of quick growth, which can be confirmed by the highest day-to-day change, which is described in the Table 1. GMV and UGMV portfolios are capable of higher returns than a benchmark, alas they also suffered higher losses, the greatest loss in a comparison to benchmark was achieved in the year 2008 as can be perceived from Figures 8 and 9. In a case of micro-level analysis GMV and UGMV portfolios performed worse than the benchmark, which can be confirmed on 95% confidence level using Student t-statistic. In a case of macro-level analysis the difference is not statistically significant; we can thus make the conclusion that in the case of the macro - level portfolio the diversification opportunities are quite limited when a dynamic portfolio formation approach was employed.

The variable volatility component had mixed impact on a portfolio performance. Probably an existence of a reliable tool, which can forecast changes of a price direction, could significantly improve outcomes of these portfolios. This can lead to synergic analysis with other methods of technical analysis, which could be further analysed.

Conclusions

In the case of LCA portfolio it was shown that macro-level portfolio performed worse than micro-level portfolio in a comparison to the benchmark, which shows that the proposed dynamic portfolio formation method has probably a greater opportunity for diversification in a case of analysis of individual shares.

The study confirms positive effects of portfolio optimization based on dynamic analysis, which is consistent with findings of Yilmaz (2010). Results in a case of LCA portfolios show promising results with a better performance than the benchmark. The DCC GARCH model proved its quality, but results of incorporating volatility components employed in GMV and UGMV portfolios are mixed. While portfolio formation based on dynamic conditional correlations showed statistically significant improvements over the benchmark, portfolio formation based on the dynamic evolution of variance based on GARCH (1,1) model did not perform so well. Thus it is probable that more complex models describing volatility processes could offer better outcomes and overall improved portfolio performance. The study shows that there is plenty of space for improvements regarding multiple criteria.

We proposed a new way of portfolio formation, which incorporated conditional correlations from the DCC GARCH model. The LCA method resulted in less volatile portfolio formation, which performed on par or even better than used benchmark. The proposed method showed that conditional correlations estimated by the DCC GARCH model can be successfully employed in portfolio formation based on GMV approach even without incorporation of volatility terms estimated GARCH (1,1) model. This offers opportunity how to simplify portfolio compositions and minimize the number of computed parameters needed in further portfolio formations.

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Kontakt

PhDr. Michael Princ
IES, FSV, Univerzita Karlova v Praze
Čs. armády 368/5
160 00 Praha 6
Česká republika
mp.princ@seznam.cz