Can we improve understanding of the financial market dependencies in the crisis by their decomposition?

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Are these two price plots different?

Randomly generated white noise process

DJI 30 index from 3th Jan. 2000 until 19th Sept. 2011
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Randomly generated white noise process

DJI 30 index from 3th Jan. 2000 until 19th Sept. 2011
Does return plot help?

White noise has constant volatility $\sigma$

DJI 30 index has time-varying volatility $\sigma_t$
Does return plot help?

White noise has constant volatility $\sigma_0$.

DJI 30 index has time-varying volatility $\sigma_t$. 

Does return plot help?
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- ... and uncertainty in volatility (risk) is nightmare for the financial world.
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- Constant volatility $\implies$ constant future risk.
- Unfortunatelly, Figure 2 represents the reality...
- ... and uncertainty in volatility (risk) is nightmare for the financial world.
- (Especially last few years)
Volatility

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- Creates the most important stylized facts (heavy-tails).
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Thus the most important question is
How do we estimate the volatility from data?
How do we estimate the volatility?

- Historical volatility PROBLEM: constant
- Exponential volatility PROBLEM: lagged
- (G)ARCH family PROBLEM: parametric, low explanatory power
- Realized Volatility (i.e. 5-minute returns) PROBLEM: need of large datasets

$$\hat{RV} = \sum_{i=1}^{m} r_i^2 \rightarrow \sigma_t$$  \hspace{1cm} (1)

where $i$ is $m$-th 5-min. return during the day.

Still, Realized Volatility seems to be best estimator

It is nonparametric and is unbiased and consistent under some assumptions
Can we find a *true* process generating the financial market series?
It seems we are really close

- Look at the unconditional distribution of returns (standardized by estimated volatility) shows that it is close to $\mathcal{N}(0,1)$. 
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- Figures show histogram and QQ plot.
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- Our contribution is in bringing volatility to time-frequency domain.
Does decomposition bring further insights?

BP futures volatility in time-frequency domain:
5-10 min
Does decomposition bring further insights?

BP futures volatility in time-frequency domain:
5-10 min, 10-20 min
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BP futures volatility in time-frequency domain:
5-10 min, 10-20 min, 20-40 min
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BP futures volatility in time-frequency domain:
5-10 min, 10-20 min, 20-40 min, 40-80 min
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BP futures volatility in time-frequency domain:
5-10 min, 10-20 min, 20-40 min, 40-80 min, up to 1-day volatility
Does decomposition bring further insights?

BP futures volatility in time-frequency domain: 5-10 min, 10-20 min, 20-40 min, 40-80 min, up to 1 day volatility and Jump variation
Does decomposition bring further insights?

BP futures volatility in time-frequency domain:
Total volatility (sum of all)
Today’s talk

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- Further, we are able to decompose it into several investment horizons.
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**Aim:** To understand the risk and correlation

In order to understand the risk, the main challenge is to understand the “\textit{true}” correlations underlying the stock markets.
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- As a HF data became available, unobservable part of stochastic volatility models, integrated variance and covariance became, observable.
- Can we improve understanding of the processes using wavelet decomposition? (Time-frequency approach).
- Can we improve forecasting and risk management?
Problem:
How to estimate the covariance from the data?

Andersen et al. (2003) using seminal result in quadratic variation theory provides a simple estimator:

Realized covariance

\[
\hat{RC}_{t,h} = \frac{1}{n} \sum_{i=1}^{n} (r_{t-h}^i + \epsilon_{t-h}^i) (r_{t-h}^{i+1} + \epsilon_{t-h}^{i+1}),
\]

where \( n \) is the number of observations in \([t-h, t]\). \(\hat{RC}_{t,h}\) is consistent and unbiased only under no noise and jumps in the data.
Estimation of “true” Covariance

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**Realized covariance**

The realized covariance over \([t - h, t]\), for \(0 \leq h \leq t \leq T\), is defined by

\[
\hat{RC}_{t,h} = \sum_{i=1}^{n} r_{t-h+i/n} r'_{t-h+i/n},
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- In this work, we use wavelets to deal with all the problems.
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Following 3 slides
- shows main theoretical results
- the very general results allow us to use the estimators...
- ... and you will trust me that there is some sound theory behind these appealing plots as well
- But, no time for explanations
Martingale representation theorem

Theorem 1
For any $m$-dimensional, square-integrable, continuous sample path, logarithmic price process $(p_t)_{t \in [0,T]}$, with a continuous sample path and a full rank of the associated $m \times m$ quadratic variation process, $[r, r]_t$, there exists a representation such that for all $0 \leq t \leq T$:

$$r_{t,h} = \int_{t-h}^{t} \mu_s ds + \int_{t-h}^{t} \Theta_s dW_s$$
Main Theoretical Result:
Martingale representation theorem by wavelets

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$$ r_{t,h} = \int_{t-h}^{t} \mu_s ds + \int_{t-h}^{t} \Theta_s dW_s $$

$$ = \int_{0}^{t} \int_{0}^{\infty} \int_{\mathbb{R}} \Psi_{j,k}(s) \langle \Psi_{j,k}, \mu_s \rangle dk \frac{1}{j^2} dj ds $$

$$ + \int_{0}^{t} \int_{0}^{\infty} \int_{\mathbb{R}} \Psi_{j,k}(s) \langle \Psi_{j,k}, \sigma_s \rangle dk \frac{1}{j^2} dj dW_s, \quad (3) $$

where $\mu_s$ is an integrable, predictable and finite-variation $m \times 1$ vector, $\Theta$ represents a multivariate stochastic volatility process with càdlàg elements, and vector $W_t$ is $m \times 1$ standard Brownian motion. $\Psi_{j,k} \in L^2(\mathbb{R}, \mathbb{R}^{m \times m})$ represents the Daubechies (D4) wavelet function with a compact support.

Proof based on Calderón reconstruction formula
Theoretical result:
Wavelet based estimator of CV cont.

The following estimator based on the MODWT of the jump-adjusted returns data $Y^{(J)}$ solves both problems.
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**Definition: Jump wavelet TSCV (JWTSCV) estimator**

$$
\widehat{RC}_{(l,q)_{t,h}}^{(JWTSCV)} = c_N \left( \widehat{RC}_{(l,q)_{t,h}}^{(WRC,J)} - \frac{nG}{n_S} \widehat{RC}_{(l,q)_{t,h}}^{(S,J)} \right), \tag{4}
$$

where

$$
\widehat{RC}_{(l,q)_{t,h}}^{(WRC,J)} = \frac{1}{G} \sum_{g=1}^{G} \sum_{j=1}^{J_s+1} \sum_{k=1}^{n} \widehat{W}_{(l)_{j,t-h+k\frac{k}{\bar{n}}}} \widehat{W}_{(q)_{j,t-h+k\frac{k}{\bar{n}}}}
$$

obtained from wavelet coefficient estimates using the MODWT on a grid of size $\bar{n} = n/G$ on the jump-adjusted observed data, $y^{(J)}_{t,h} = y_{t,h} - \widehat{MWJC}$, and $c_N$ is a constant that can be tuned for small sample performance.
Theoretical result:
Wavelet based estimator of CV cont.

- J-WTSCV is unbiased estimator of $CV_{t,h}$.
- J-WTSCV is consistent estimator of $CV_{t,h}$.

(Proofs provided upon request).

Thus J-WTSCV is able to consistently estimate $CV_{t,h}$ in the presence of jumps and noise.

Moreover, J-WTSCV provides decomposition of true volatility into investment horizons.

Main Advantage

The idea is very simple (simple things tend to work!)

Although the theory looks rather complicated

Estimator is actually very simple, everyone can use it i.e. with Excel.
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- Until now, no-one have shown properly the properties of multivariate returns. (difficult task)
- Our main results?
  - We decompose the multivariate volatility into jumps, co-jumps, deal with the noise.
  - We further decompose it to several investment horizons.
  - Thus we bring a new way of understanding of the dependence...
  - ... with simple methodology.
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Applications: Decomposition of stock market dynamics

Figure: A “True” dependence of the GBP-EUR futures pair decomposed to several investment horizons, Jumps and Co-Jumps
Applications: Dynamic correlations estimation

- Once we have decomposition, we can easily compute dynamic correlations (very precisely):
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- Once we have decomposition, we can easily compute dynamic correlations (very precisely):

![Graph: Dynamic correlation of GBP-EUR futures pair with 95% confidence interval.](image)

**Figure:** Dynamic correlation of GBP-EUR futures pair with 95% confidence interval.
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- Significant improvement in estimation and forecasting of portfolio beta.
- Decomposition of all the variables.
Conclusion

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- Allow for great improvement of correlation forecasts.
- Understanding of jumps and co-jumps.
- Finally, our results has important implications to financial world.

Discussion

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