Can we improve understanding of the financial market dependencies in the crisis by their decomposition?

Jozef Baruník

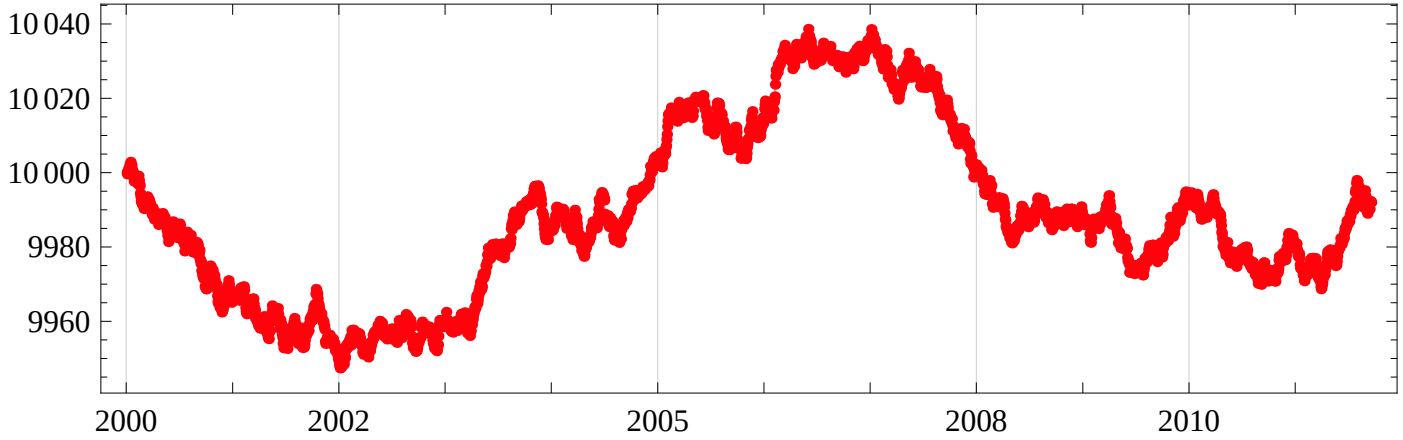
International scientific conference Czech and World Economies after the Global Financial Crisis

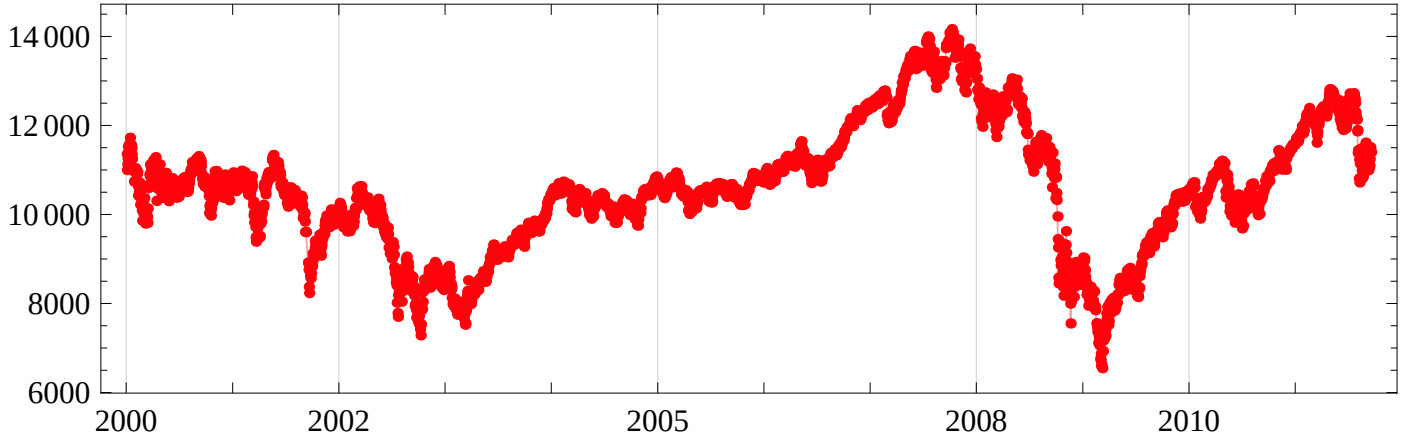
prof. F. Vencovsky Prize competiton

Congress Centre of the Czech National Bank

25.11. 2011

Are these two price plots different?





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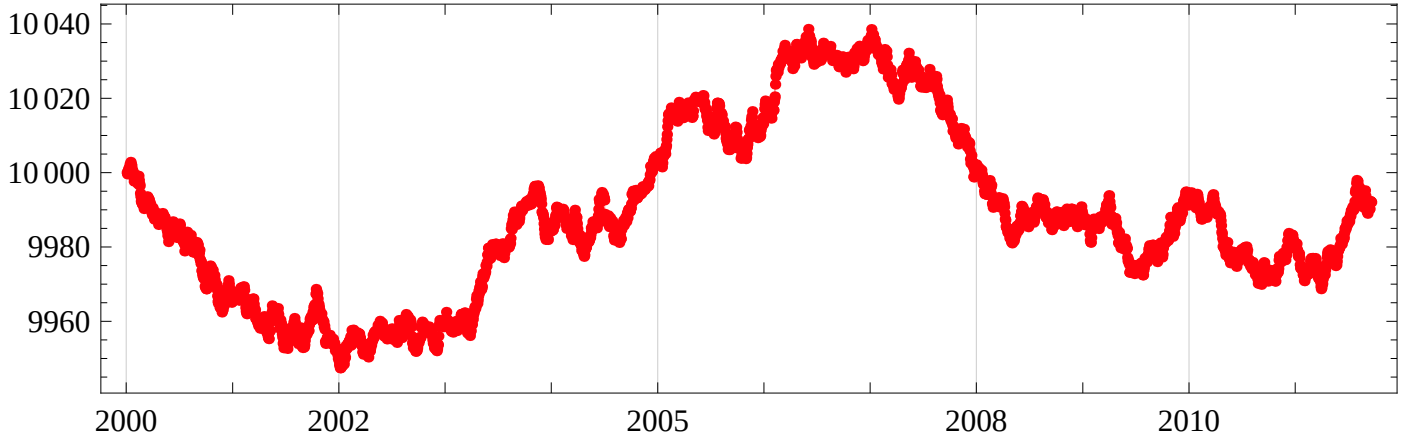
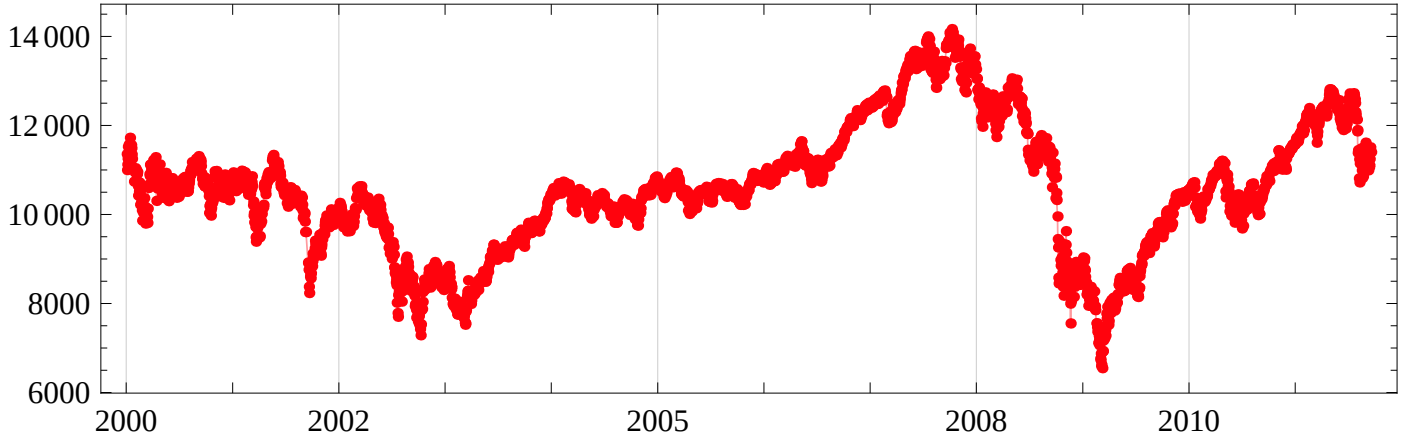
Randomly generated white noise process





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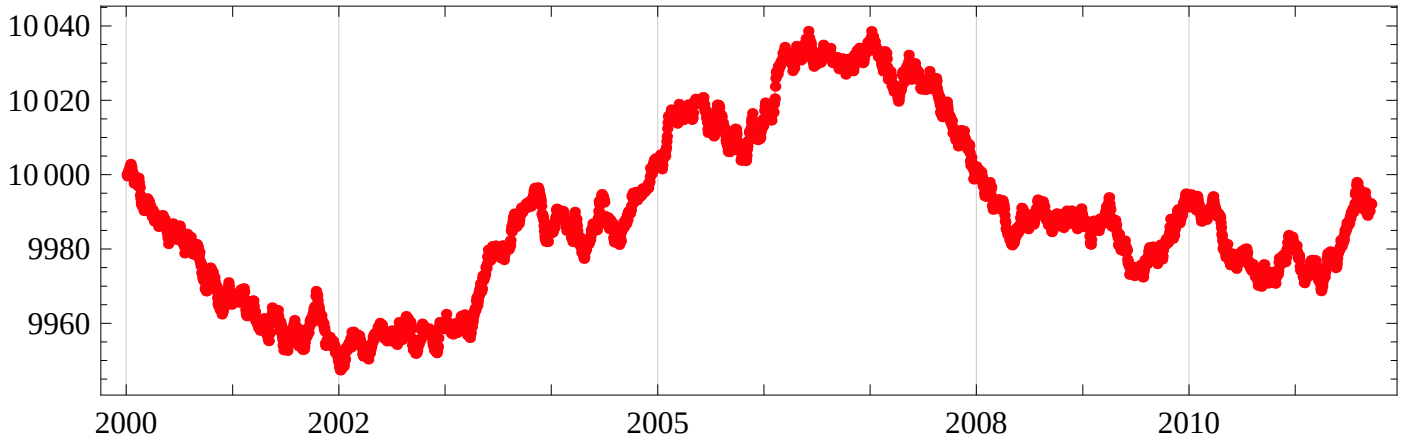
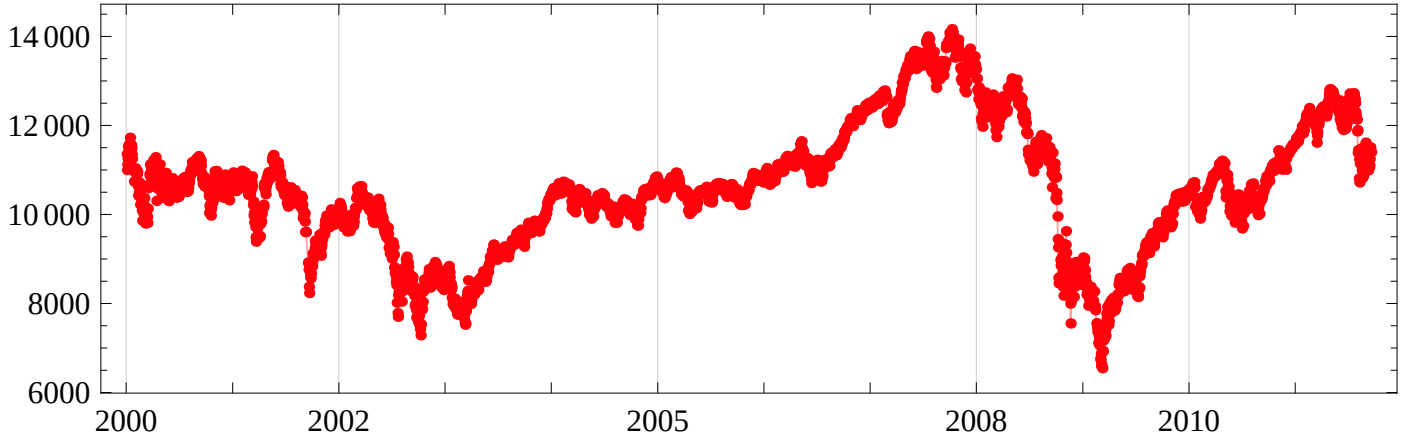


DJI 30 index from 3th Jan. 2000 until 19th Sept. 2011



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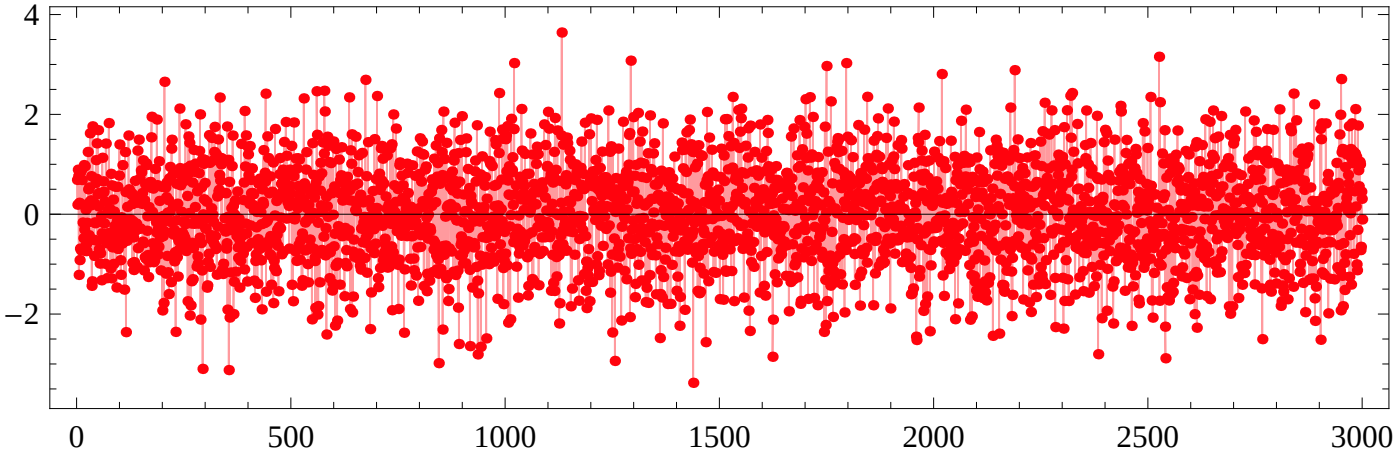

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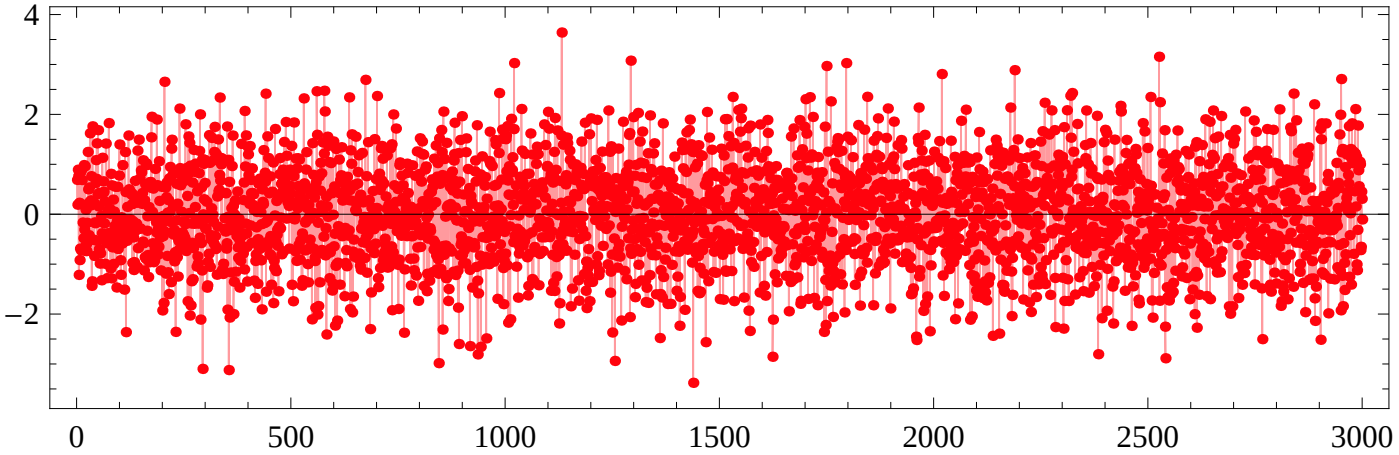

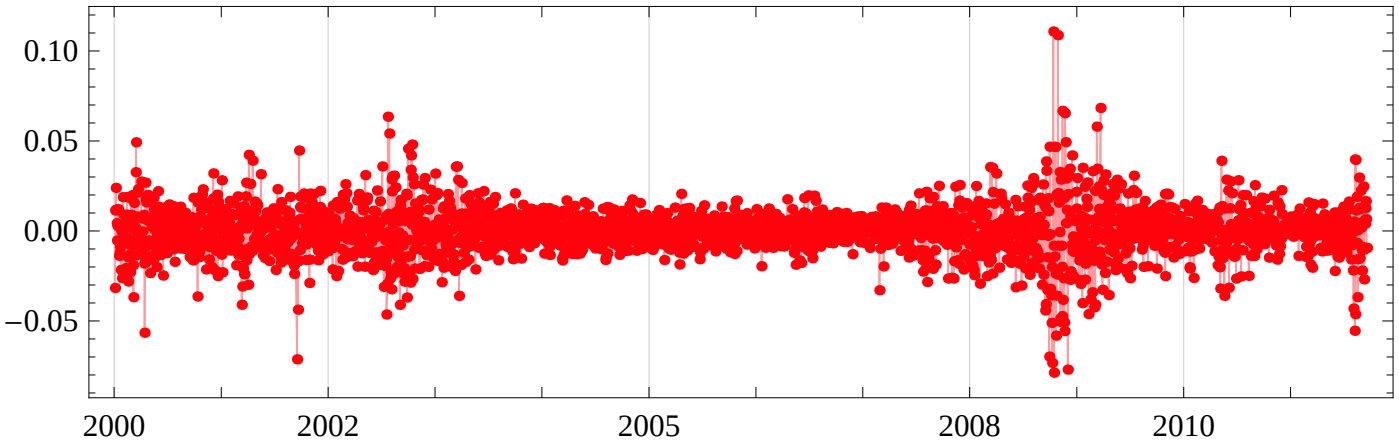

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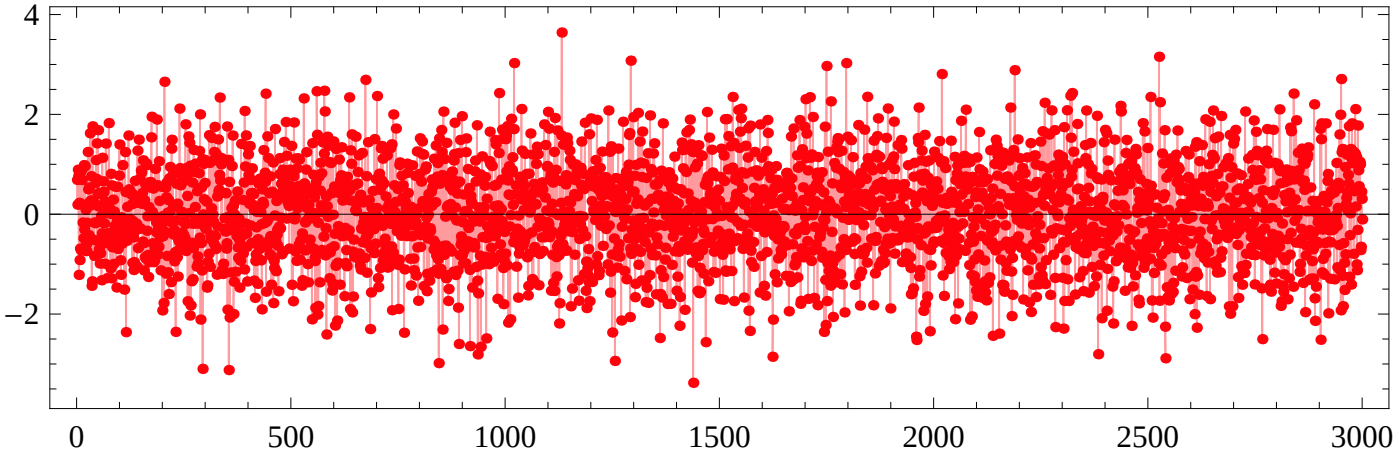
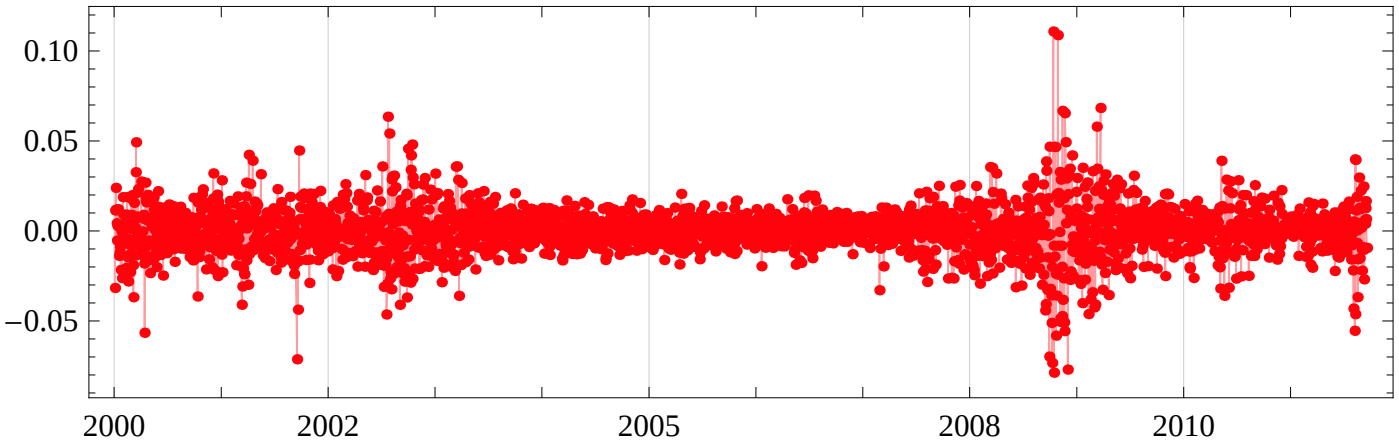
White noise has constant volatility σ





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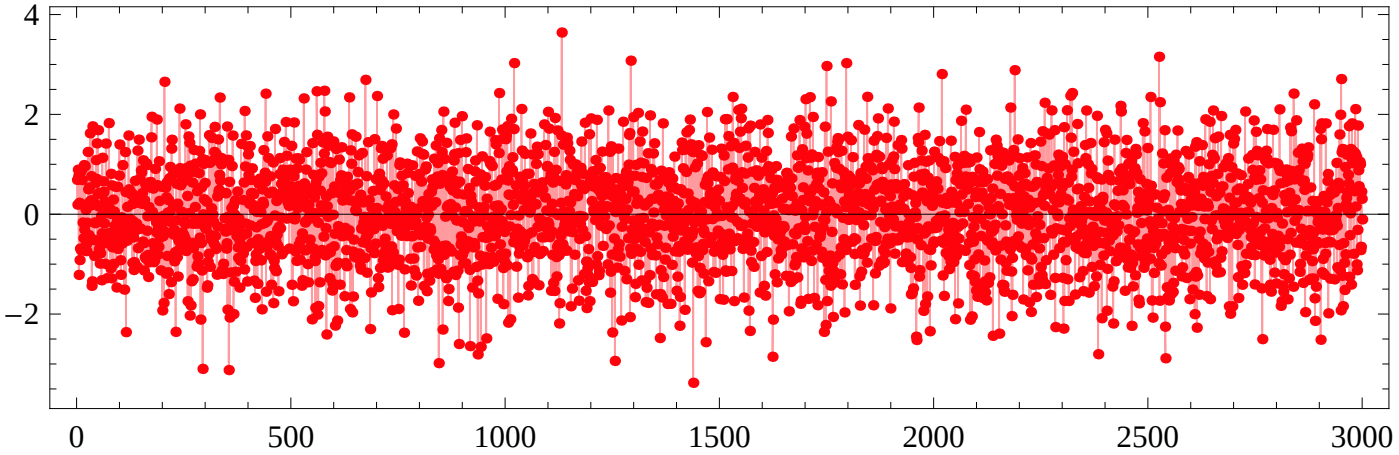
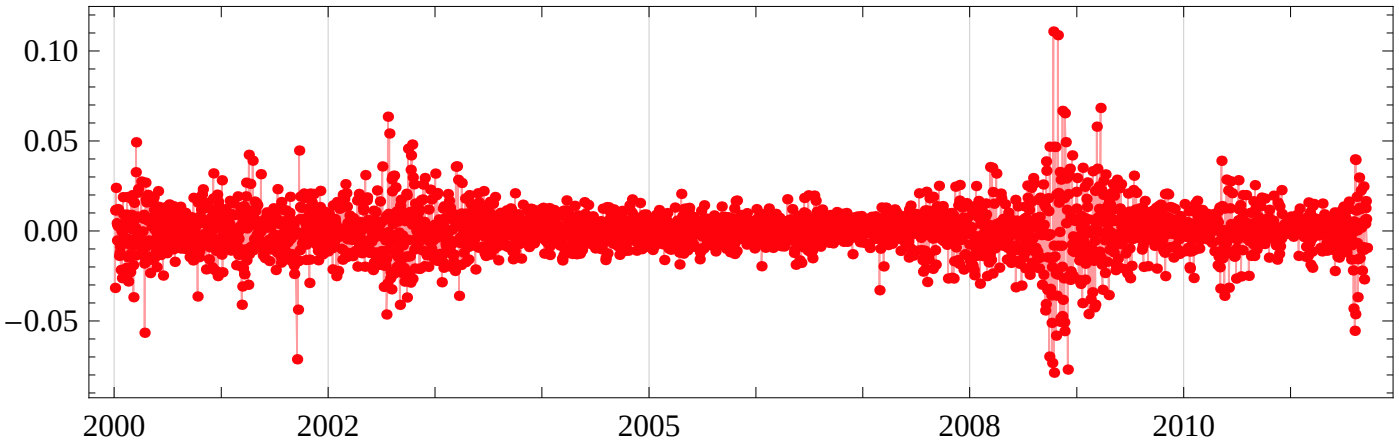


DJI 30 index has time-varying volatility σ_t



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- ... and uncertainty in volatility (risk) is nightmare for the financial world.
- (Especially last few years)

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Thus the most important question is

How do we estimate the volatility from data?

How do we estimate the volatility?

- Historical volatility PROBLEM: constant
- Exponential volatility PROBLEM: lagged
- (G)ARCH family PROBLEM: parametric, low explanatory power
- Realized Volatility (i.e. 5-minute returns) PROBLEM: need of large datasets

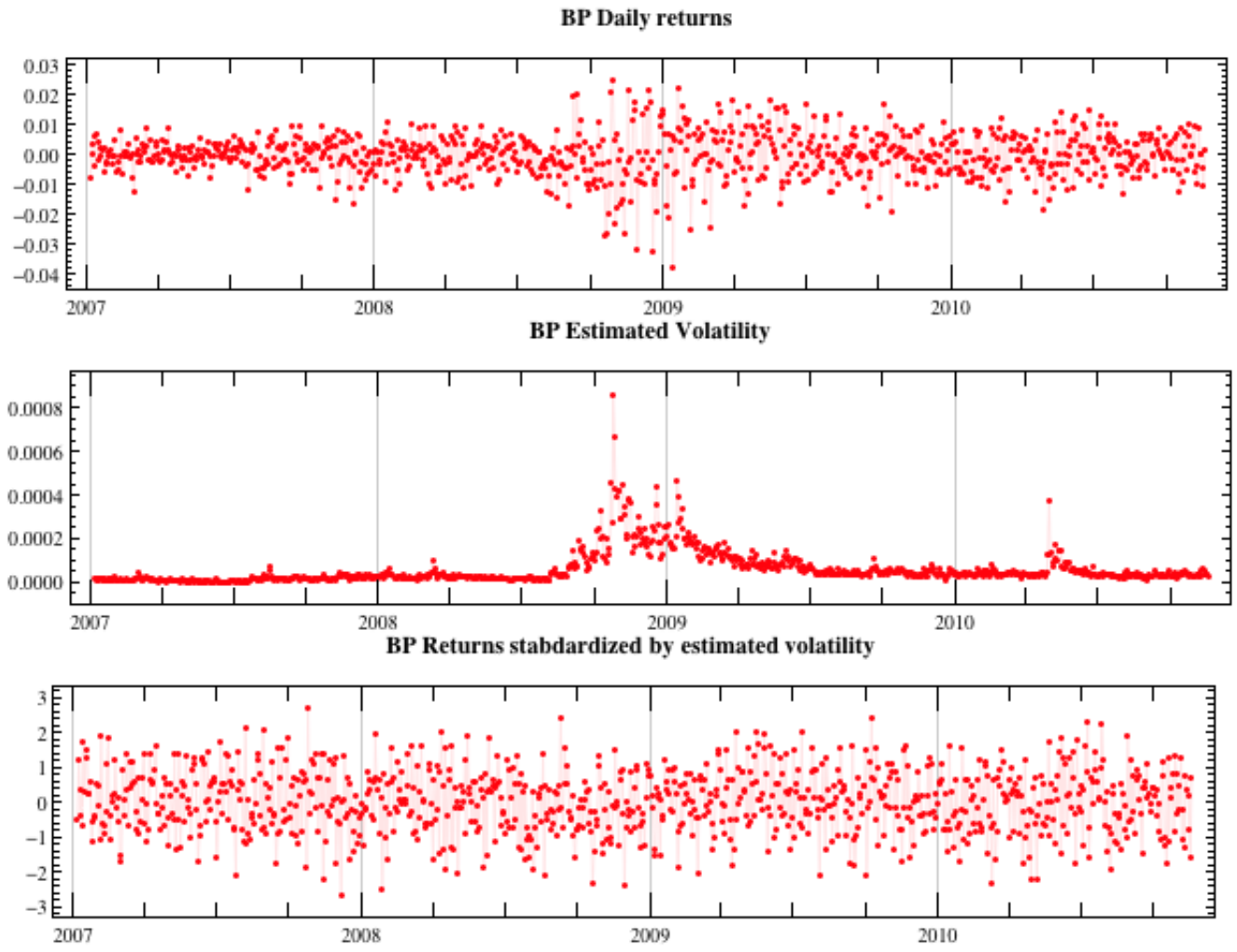
$$\widehat{RV} = \sum_{i=1}^{m} r_i^2 \rightarrow \sigma_t \tag{1}$$

where i is m-th 5-min. return during the day.

Still, Realized Volatility seems to be best estimator

It is nonparametric and is unbiased and consistent under some assumptions

Can we find a *true* process generating the financial market series?



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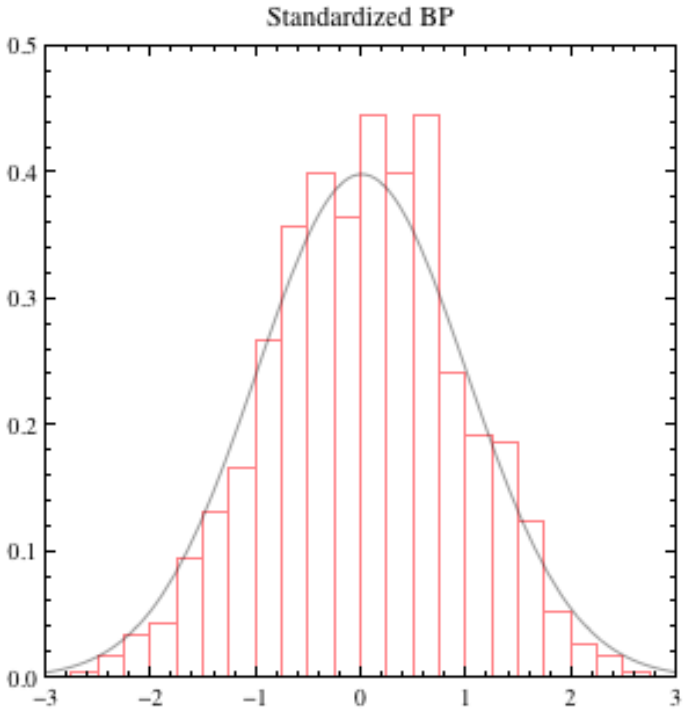

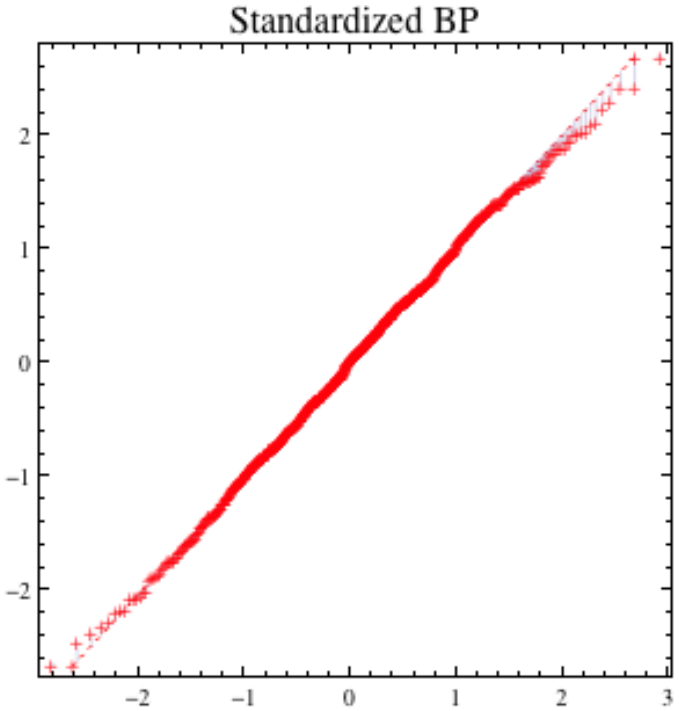

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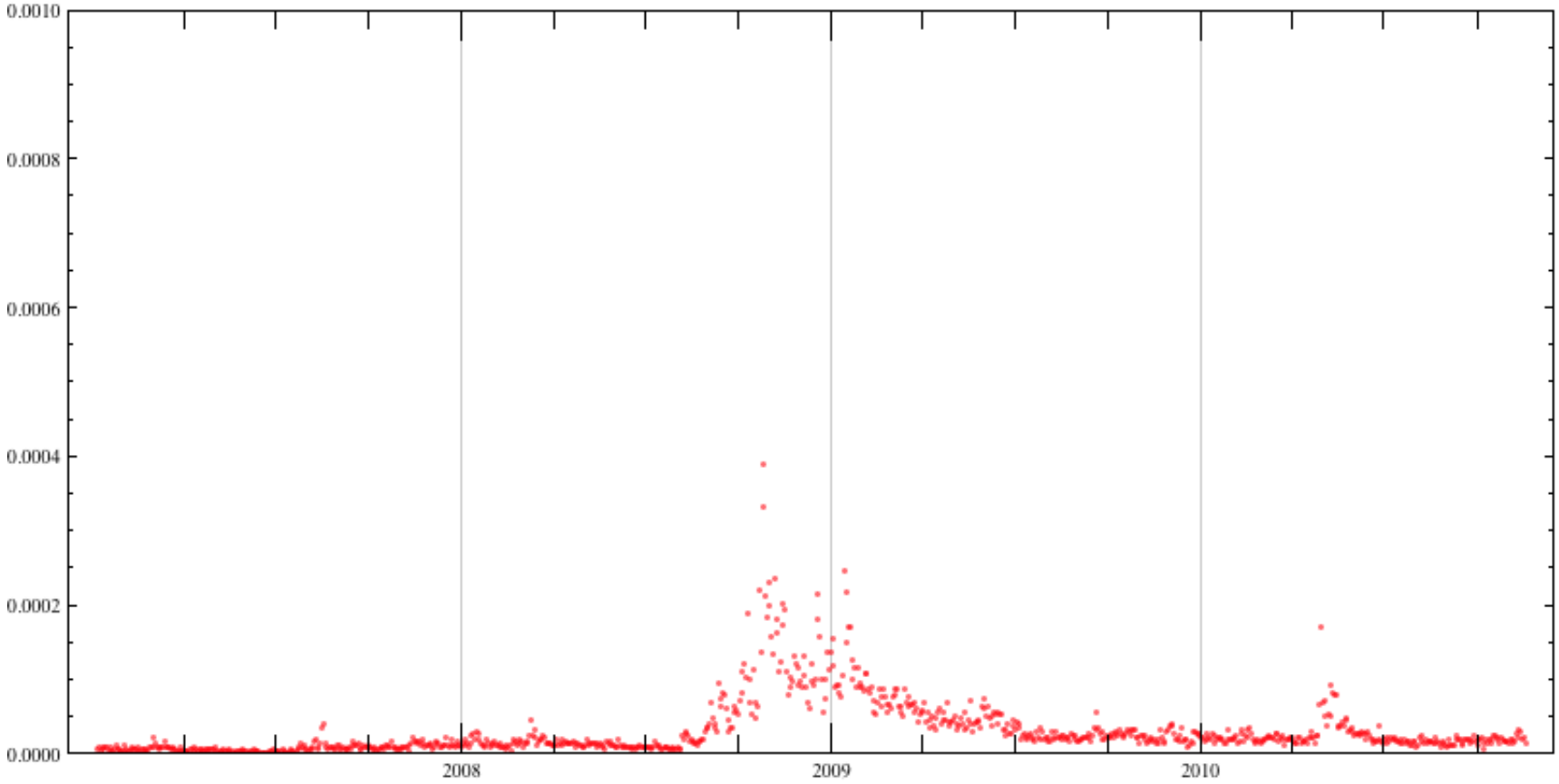
- Our contribution is in bringing volatility to time-frequency domain.

Does decomposition bring further insights?

BP futures volatility in time-frequency domain:
5-10 min



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BP futures volatility in time-frequency domain:
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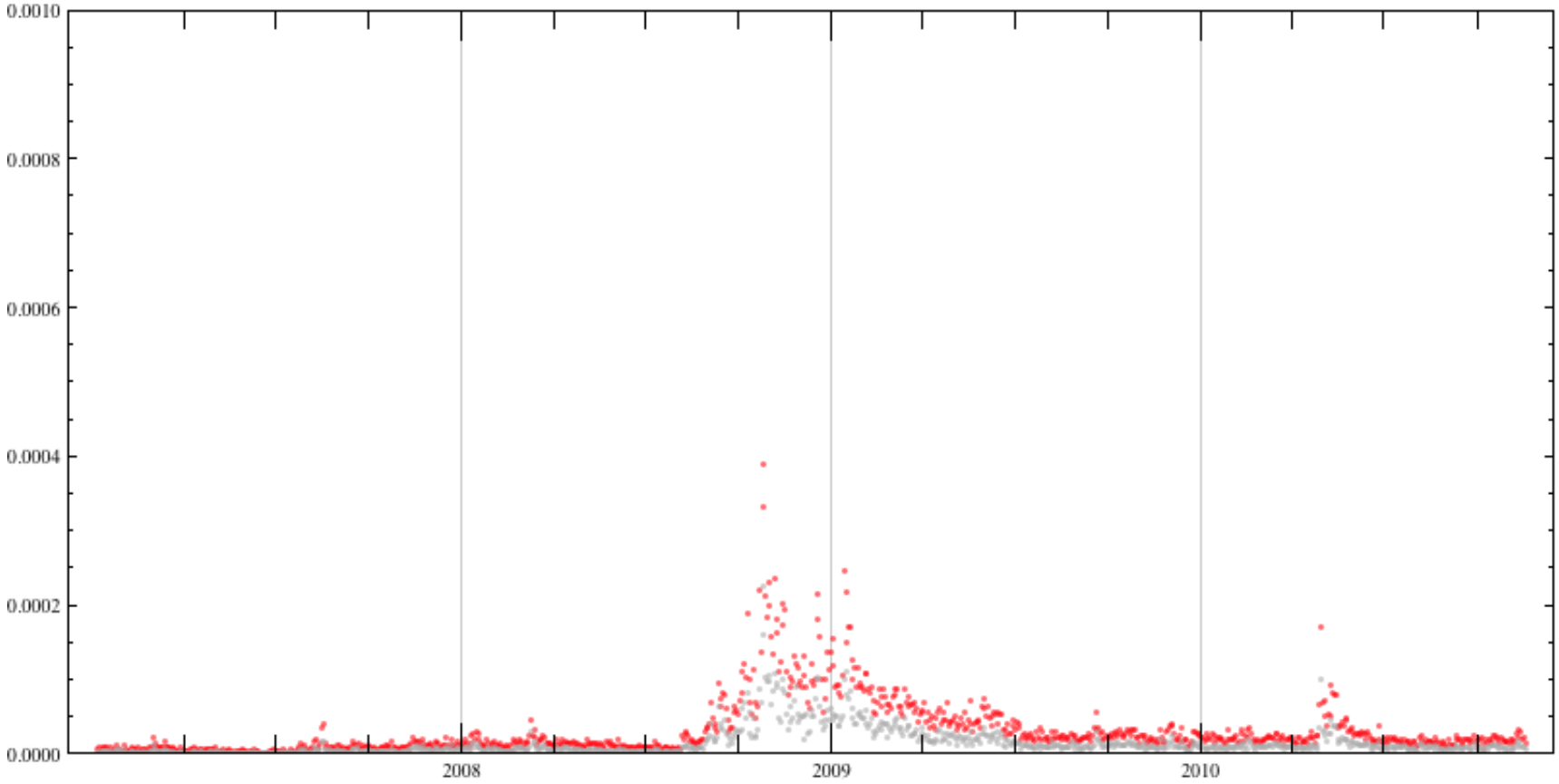

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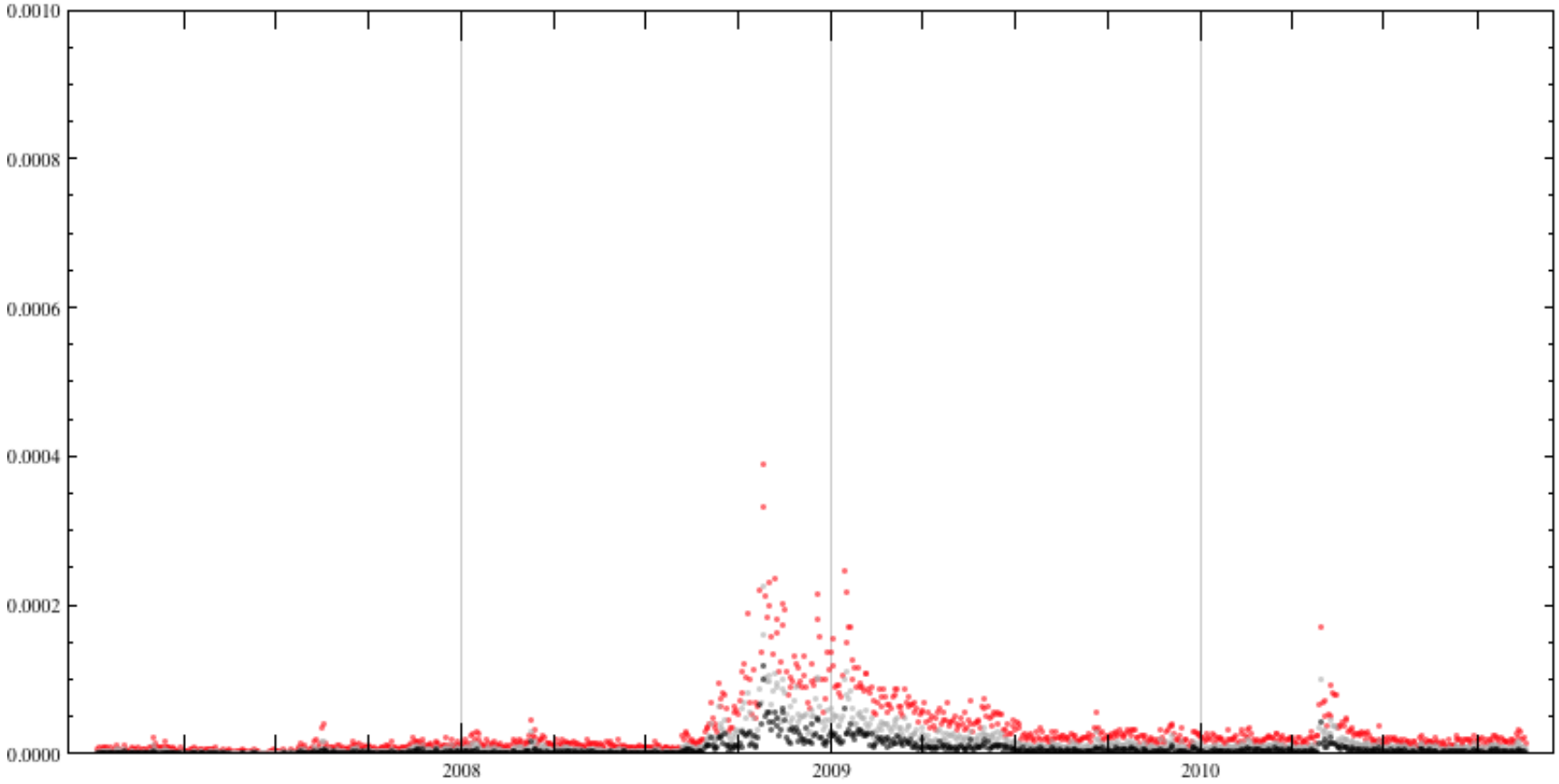

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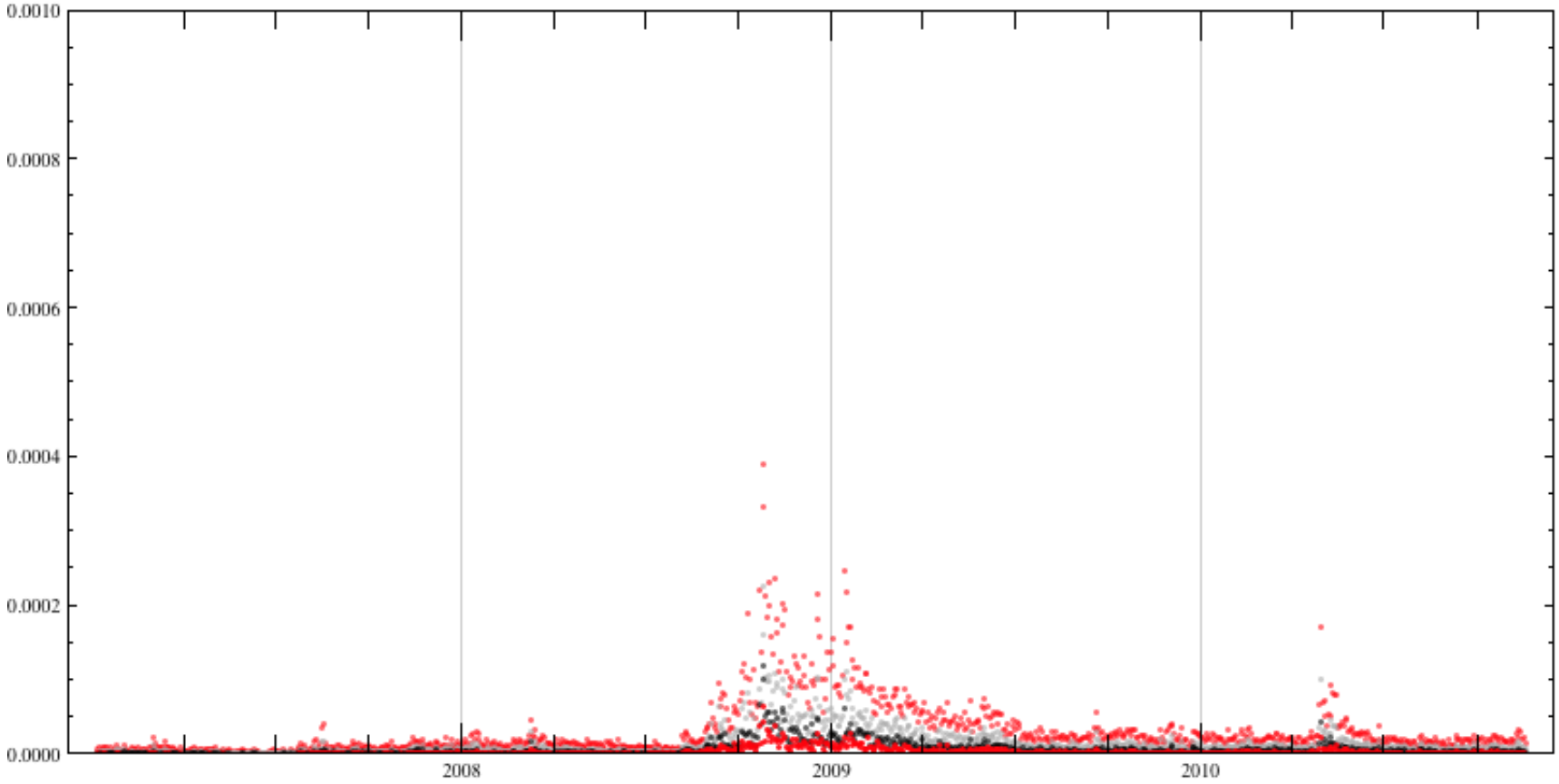

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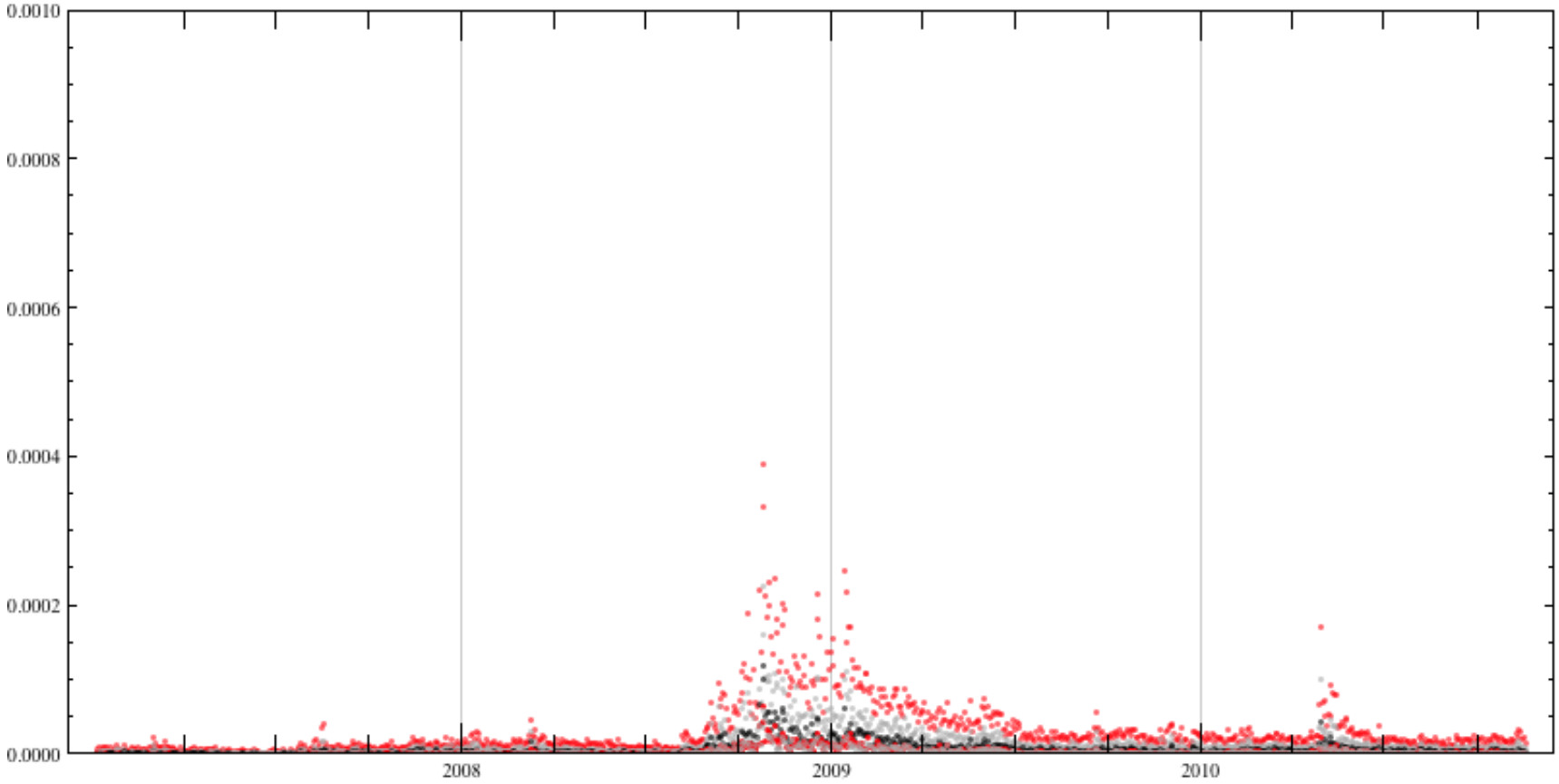
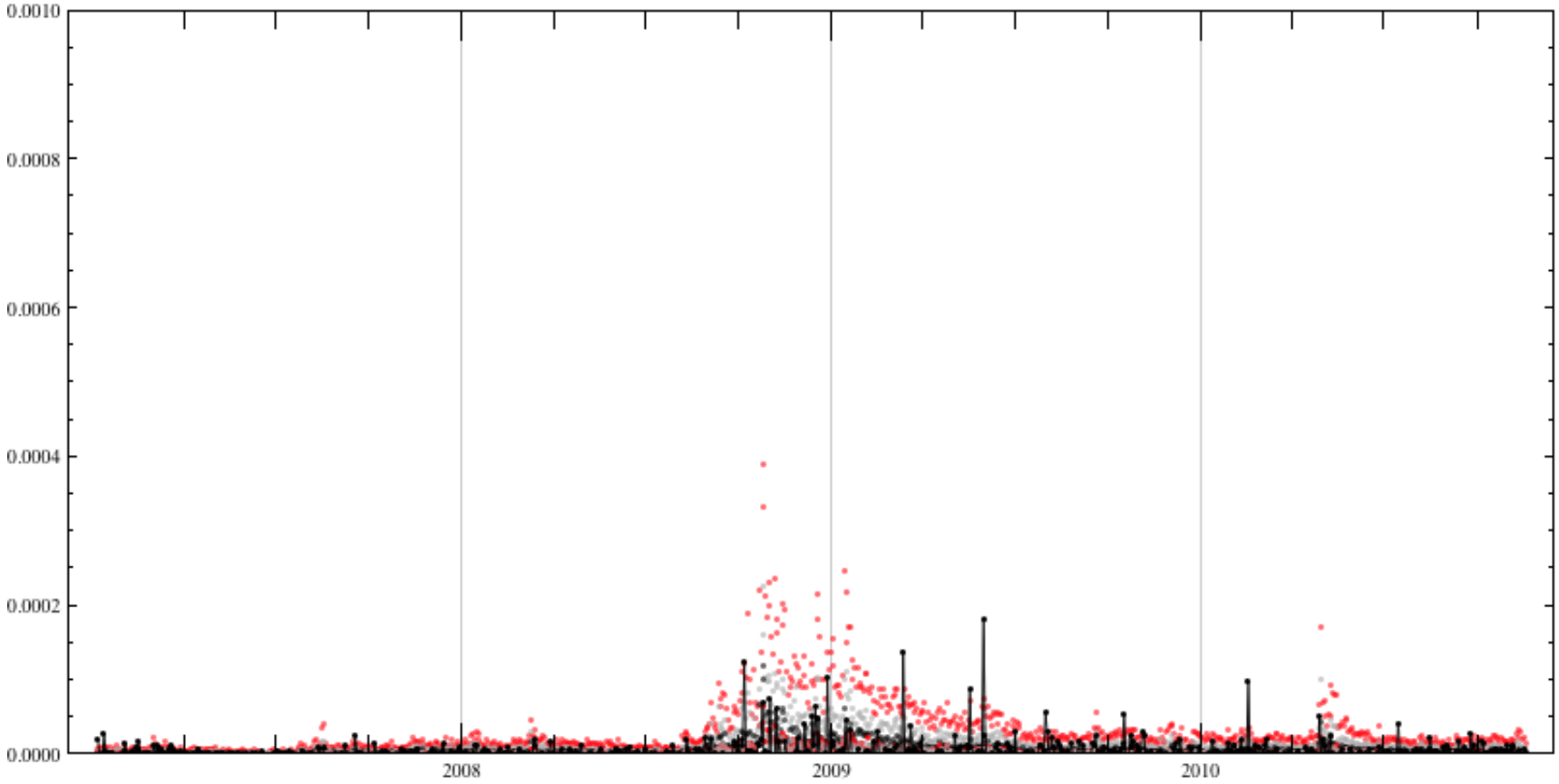


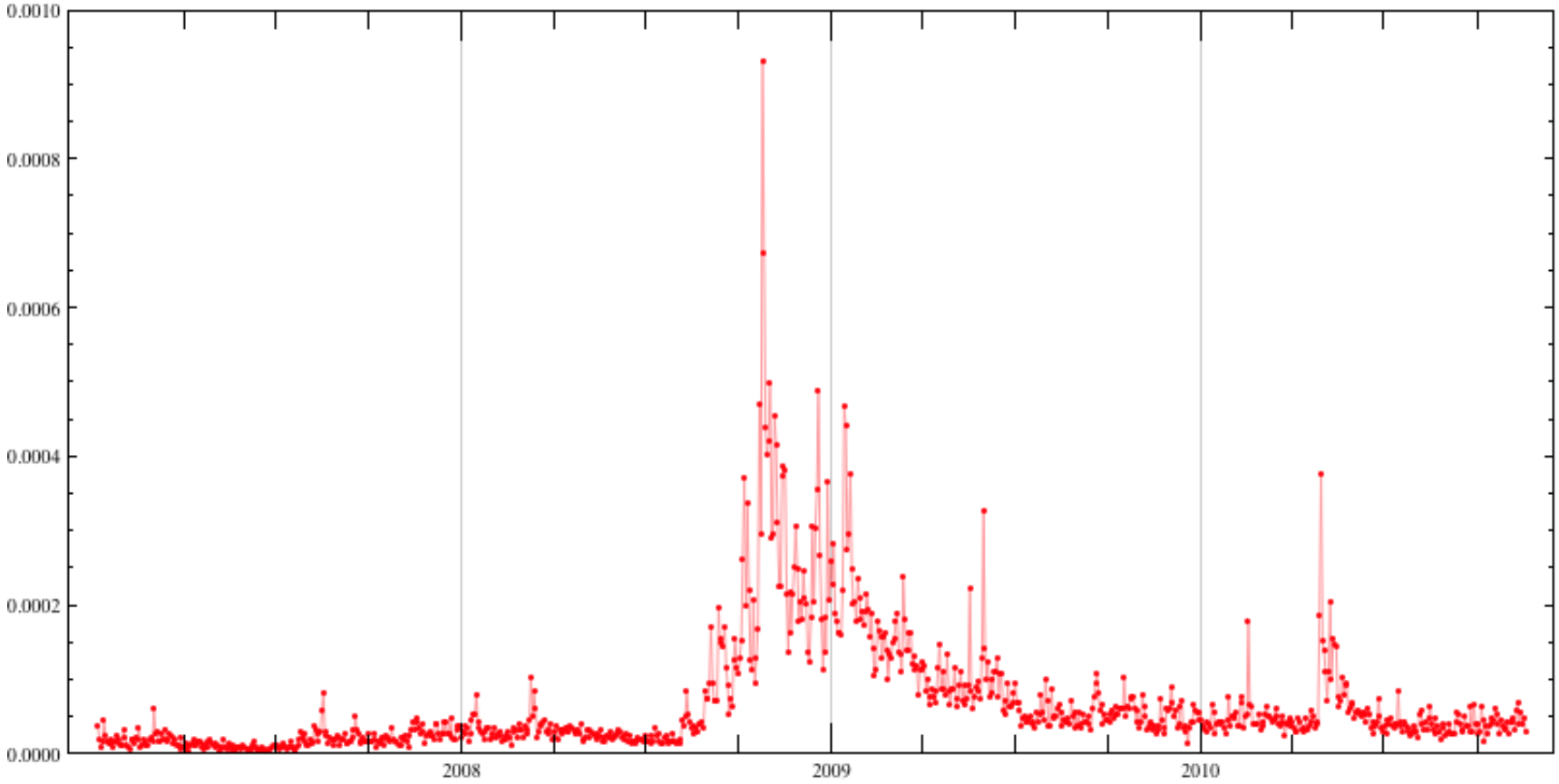
Does decomposition bring further insights?

BP futures volatility in time-frequency domain:
5-10 min, 10-20 min, 20-40 min, 40-80 min, up to 1day volatility and Jump variation



Does decomposition bring further insights?

BP futures volatility in time-frequency domain:
Total volatility (sum of all)



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Aim: To understand the risk and correlation

In order to understand the risk, the main challenge is to understand the *"true"* correlations underlying the stock markets.

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- Can we improve understanding of the processes using wavelet decomposition? (Time-frequency approach).
- Can we improve forecasting and risk management?

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Realized covariance

The realized covariance over $[t-h, t]$, for $0 \leq h \leq t \leq T$, is defined by

$$\widehat{RC}_{t,h} = \sum_{i=1}^{n} \mathbf{r}_{t-h+\left(\frac{i}{n}\right)h} \mathbf{r}'_{t-h+\left(\frac{i}{n}\right)h}, \tag{2}$$

where n is the number of observations in $[t-h, t]$.

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- $\widehat{RC}_{t,h}$ is consistent and unbiased only under no noise and jumps in the data.

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Following 3 slides

- shows main theoretical results
- the very general results allow us to use the estimators...
- ... and you will trust me that there is some sound theory behind these appealing plots as well
- But, no time for explanations

Martingale representation theorem

Theorem 1

For any m-dimensional, square-integrable, continuous sample path, logarithmic price process $(\mathbf{p}_t)_{t\in[0,T]}$, with a continuous sample path and a full rank of the associated $m \times m$ quadratic variation process, $[r, r]_t$, there exists a representation such that for all $0 \leq t \leq T$:

$$\mathbf{r_{t,h}} = \int_{t-h}^{t} \boldsymbol{\mu_s} ds + \int_{t-h}^{t} \boldsymbol{\Theta_s} d\boldsymbol{W_s}$$

Main Theoretical Result: Martingale representation theorem by wavelets

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$$\begin{aligned}
\mathbf{r_{t,h}} &= \int_{t-h}^{t} \boldsymbol{\mu_s} ds + \int_{t-h}^{t} \boldsymbol{\Theta_s} d\boldsymbol{W_s} \\
&= \int_0^t \int_0^\infty \int_{\mathbb{R}} \boldsymbol{\Psi_{j,k}(s)} \langle \boldsymbol{\Psi_{j,k}}, \boldsymbol{\mu_s} \rangle dk \frac{1}{j^2} dj ds \\
&\quad + \int_0^t \int_0^\infty \int_{\mathbb{R}} \boldsymbol{\Psi_{j,k}(s)} \langle \boldsymbol{\Psi_{j,k}}, \boldsymbol{\sigma_s} \rangle dk \frac{1}{j^2} dj d\boldsymbol{W_s}, \qquad (3)
\end{aligned}$$

where $\boldsymbol{\mu}_s$ is an integrable, predictable and finite-variation $m \times 1$ vector, $\boldsymbol{\Theta}$ represents a multivariate stochastic volatility process with càdlàg elements, and vector $\boldsymbol{W_t}$ is $m \times 1$ standard Brownian motion. $\boldsymbol{\Psi_{j,k}} \in L^2(\mathbb{R}, \mathbb{R}^{m\times m})$ represents the Daubechies (D4) wavelet function with a compact support.

Proof based on Calderón reconstruction formula

Theoretical result: Wavelet based estimator of CV cont.

- The following estimator based on the MODWT of the jump-adjusted returns data $Y^{(J)}$ solves both problems.

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Definition: Jump wavelet TSCV (JWTSCV) estimator

$$\widehat{RC}^{(JWTSCV)}_{(l,q)t,h} = c_N \left(\widehat{RC}^{(WRC,J)}_{(l,q)t,h} - \frac{\bar{n_G}}{n_S} \widehat{RC}^{(S,J)}_{(l,q)t,h} \right), \qquad (4)$$

where
$\widehat{RC}^{(WRC,J)}_{(l,q)t,h} = \frac{1}{G} \sum_{g=1}^{G} \sum_{j=1}^{J_s+1} \sum_{k=1}^{n} \widetilde{\mathcal{W}}_{(l)j,t-h+\frac{k}{n}h} \widetilde{\mathcal{W}}_{(q)j,t-h+\frac{k}{n}h}$
obtained from wavelet coefficient estimates using the MODWT on a grid of size $\bar{n} = n/G$ on the jump-adjusted observed data, $\mathbf{y}^{(J)}_{t,h} = \mathbf{y}_{t,h} - \widehat{\mathbf{MWJC}}$, and c_N is a constant that can be tuned for small sample performance.

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- ... with simple methodology.

Applications: Decomposition of stock market dynamics

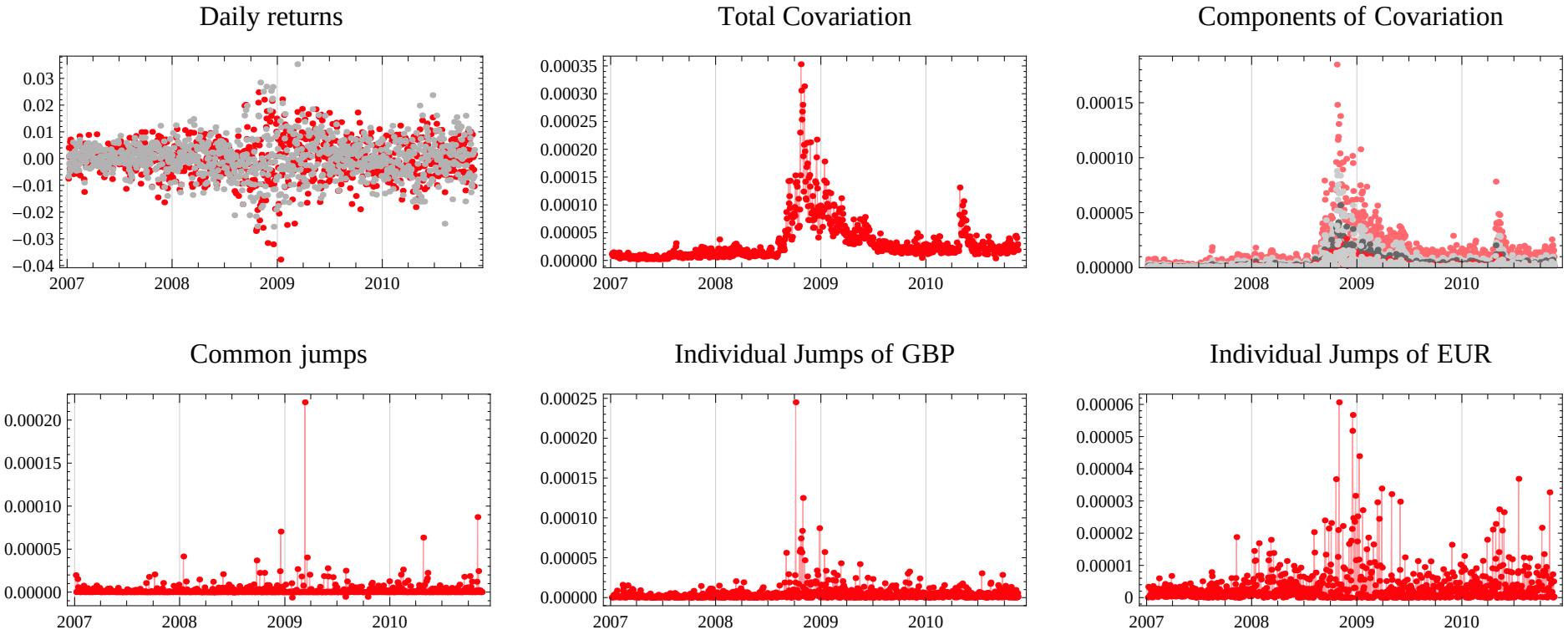

Figure: A *"True"* dependence of the GBP-EUR futures pair decomposed to several investment horizons, Jumps and Co-Jumps

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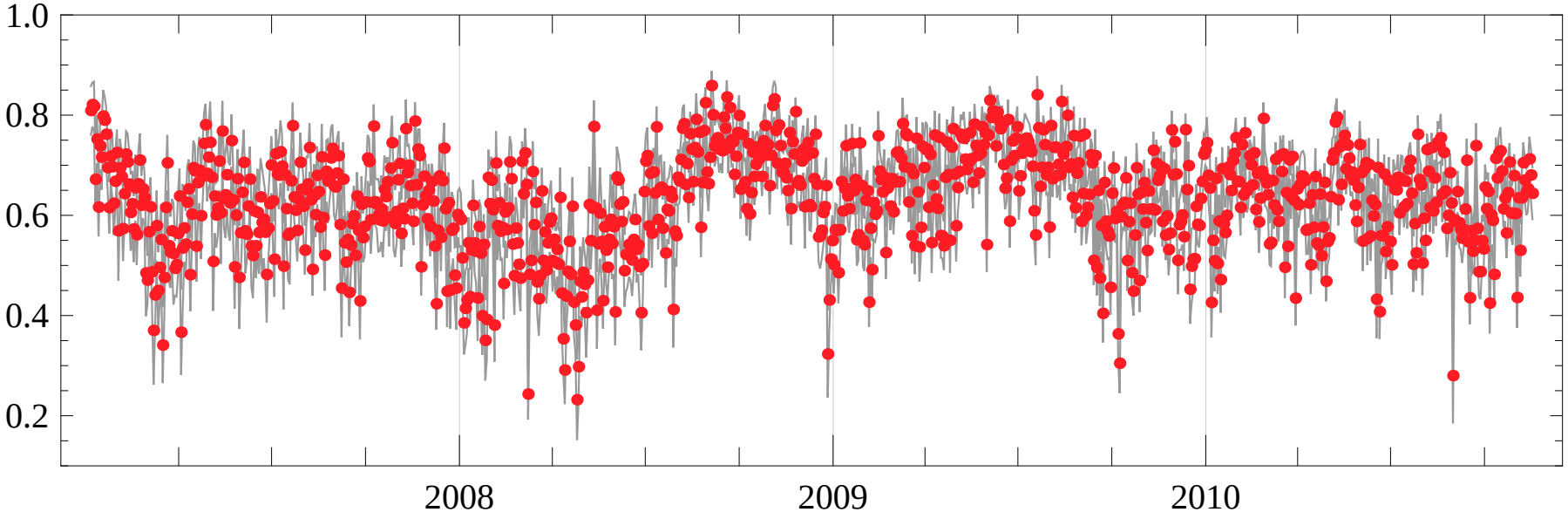

Figure: Dynamic correlation of GBP-EUR futures pair with 95% confidence interval.

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- Decomposition of all the variables.

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- Allow for great improvement of correlation forecasts.
- Understanding of jumps and co-jumps.
- Finally, our results has important implications to financial world

Discussion

Thank you very much for your attention!